Quiz #9; Wed, 3/30/2016

Math 53 with Prof. Stankova

Section 110; MWF12-1

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Problem. Use Lagrange multipliers to find the maximum and minimum values of $f(x, y) = x^2y$ subject to the constraint $x^2 + y^2 = 6$.

Solution. Let $g(x,y) = x^2+y^2-6$ (so that the constraint is $g = 0$). We compute that $\nabla f = \langle 2xy, x^2\rangle$ and $\nabla g = \langle 2x, 2y\rangle$. Thus, the method of Lagrange multipliers gives the equation:

$$\langle 2xy, x^2\rangle = \lambda\langle x, y\rangle$$

(Note that we just absorbed 2 in the $\nabla g = 2\langle x, y\rangle$ term into the λ term). Since $x^2 = \lambda y$, multiplying y to $2xy = \lambda x$, we get $2xy^2 = x^3$, so that we have $x(x^2 - 2y^2) = 0$. Since $x^2 + y^2 = 6$, we further have $x(x^2 - 2(6 - x^2)) = x(3x^2 - 12) = 3x(x^2 - 4) = 0$. So $x = -2, 2,$ or 0. So the possible triple (x, y, λ) are:

$$(\pm 2, \pm\sqrt{2}, \pm 2\sqrt{2}) \quad \text{and} \quad (0, \pm\sqrt{6}, 0)$$

In particular, the maximum and minimum of f is: $2^2\sqrt{2} = 4\sqrt{2}$ and $-4\sqrt{2}$ (respectively).