Quiz #9; Wed, 3/30/2016

Math 53 with Prof. Stankova

Section 107; MWF10-11

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Problem. Use Lagrange multipliers to find the maximum and minimum values of $f(x, y) = xyz$ subject to the constraint $x^2 + y^2 + z^2 = 6$.

Solution. Let $g(x, y, z) = x^2 + y^2 + z^2 - 6$ (so $g = 0$ is the constraint). We compute that $\nabla f = \langle yz, zx, xy \rangle$ and $\nabla g = \langle 2x, 2y, 2z \rangle$ so that the Lagrange multiplier method gives the equation:

$$\langle yz, zx, xy \rangle = \lambda \langle x, y, z \rangle$$

Since $yz = \lambda x$, we have $xyz = \lambda x^2$. Likewise, we have $xyz = \lambda y^2 = \lambda z^2$. Thus, $\lambda x^2 = \lambda y^2 = \lambda z^2$. If $\lambda \neq 0$, then $x^2 = y^2 = z^2$ so that $g = 0$ implies $x, y, z = \pm\sqrt{2}$. If $\lambda = 0$, then at least one of x, y, z (in fact, at least two) is 0, so that $f = 0$. Thus, maximum of f is $(\sqrt{2})^3 = 2\sqrt{2}$ and the minimum is $-2\sqrt{2}$.