

**Quiz #6; Wed, 3/2/2016**

**Math 53 with Prof. Stankova**

**Section 107; MWF10-11**

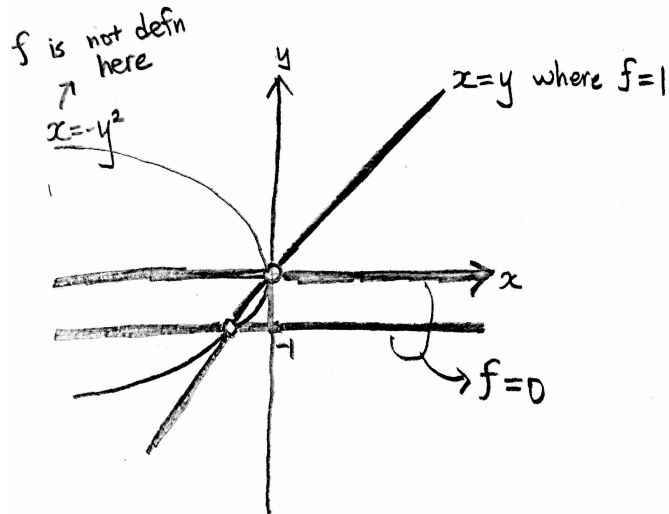
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**Student Name:** \_\_\_\_\_

*Problem.* Consider the function  $f(x, y) = \frac{y^2 + y}{x + y^2}$

- (a) Where is  $f$  NOT defined? Sketch these “bad points” on  $xy$ -plane. What is the domain of  $f$ ? (5 points)
- (b) Draw the contour curves for  $f(x, y) = 0$  and  $f(x, y) = 1$  (i.e. do the  $k = 0$  and  $k = 1$  case). Please note that since  $f(x, y)$  is not defined on the whole  $\mathbb{R}^2$ , there might be “holes” on some contour curves. *Be sure to mark these holes.* (8 points)
- (c) Prove that  $f(x, y)$  CANNOT be extended continuously to the whole plane. (It suffices to explain why  $f(x, y)$  CANNOT be extended over just one particular “bad” point). (2 points)

*Solution.* (a) & (b): Note that the function is defined (and is continuous) everywhere except when the denominator  $x + y^2 = 0$ . The points where  $x + y^2 = 0$  is drawn below. The domain is thus  $\{(x, y) \in \mathbb{R}^2 : x + y^2 \neq 0\}$ . For the contours,  $f = 0$  if and only if  $y^2 + y = 0$ , i.e. when  $y = 0$  or  $y = -1$ .  $f = 1$  when  $y^2 + y = x + y^2$ , i.e.  $x = y$ . The whole time, we keep in mind that  $x \neq -y^2$ .



(c) No. If we were to do so, the contour lines for  $k = 1$  and  $k = 0$  would overlap.