Quiz #3; Wed, 2/17/2016

Math 53 with Prof. Stankova

Section 110; MWF12-1

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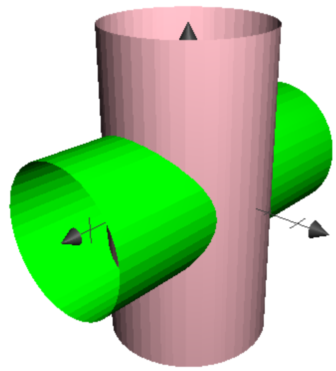
Problem. Find the equation for the tangent line to the curve of intersection of the cylinders $x^2+y^2 = 25$ and $y^2 + z^2 = 20$ at the point $(3, 4, 2)$.

Solution. First sketch the two cylinders:



(Pink is $x^2 + y^2 = 25$, and the green is $y^2 + z^2 = 20$). We are concerned with the point $(3, 4, 2)$, so we only need consider the parametrization with x-coordinate being positive. The points lie on the green cylinder entirely, so set $y = \sqrt{20}\cos t, z = \sqrt{20}\sin t$. then $x = \sqrt{25 - y^2} = \sqrt{25 - 20\cos^2 t}$. In other words, we have

$$\mathbf{r}(t) = \langle \sqrt{25 - 20\cos^2 t}, \sqrt{20}\cos t, \sqrt{20}\sin t \rangle$$

so that

$$\mathbf{r}'(t) = \langle \frac{1}{\sqrt{25 - 20\cos^2 t}}(20\cos t \sin t), -\sqrt{20}\sin t, \sqrt{20}\cos t \rangle$$

So, at the $(3, 4, 2)$, which occurs when $\cos t = \frac{4}{\sqrt{20}}, \sin t = \frac{2}{\sqrt{20}}$ (note that $\cos^2 t + \sin^2 t = 1$ here so such t does exist), the tangent vector direction is $\langle 8/3, -2, 4 \rangle$, so the tangent line is:

$$\frac{x - 3}{8/3} = \frac{y - 4}{-2} = \frac{z - 2}{4}$$

Alternatively, one can parameterize as:

$$\mathbf{r}(t) = \langle \sqrt{25 - t^2}, t, \sqrt{20 - t^2} \rangle$$

so that

$$\mathbf{r}'(t) = \langle \frac{-t}{\sqrt{25 - t^2}}, 1, \frac{-t}{\sqrt{20 - t^2}} \rangle$$

at $t = 4$ to get $\langle -4/3, 1, -2 \rangle$, which gives the same line.