Quiz #4; Wed, 2/17/2016

Math 53 with Prof. Stankova

Section 107; MWF10-11

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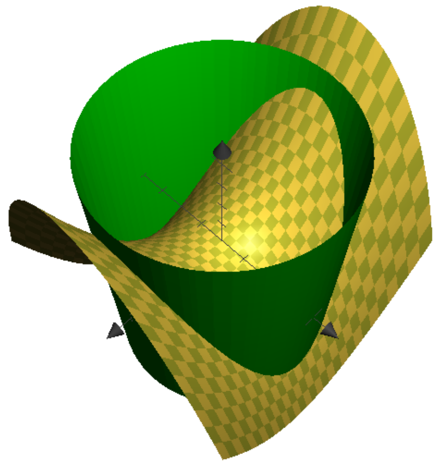
Problem. Find an equation for the tangent line to the curve of intersection of the hyperbolic paraboloid $z = x^2 - y^2$ and the cylinder $x^2 + y^2 = 1$ at the point $(\frac{\sqrt{3}}{2}, \frac{1}{2}, \frac{1}{2})$

Solution. Let's sketch the surfaces:



(The green is the cylinder, and the yellow is the hyperbolic paraboloid).

For the curve of intersection, we can parameterize it as:

$$\mathbf{r}(t) = \langle \cos t, \sin t, \cos^2 t - \sin^2 t \rangle$$

so that (noting $\cos^2 t - \sin^2 t = \cos(2t)$)

$$\mathbf{r}(t) = \langle -\sin t, \cos t, -2\sin(2t) \rangle$$

At $t = \pi/6$, which is when $\mathbf{r}(t) = (\frac{\sqrt{3}}{2}, \frac{1}{2}, \frac{1}{2})$, we have $\mathbf{r}'(\pi/6) = \langle -\frac{1}{2}, \frac{\sqrt{3}}{2}, -\sqrt{3} \rangle$. Hence, one way to write the equation for the tangent line is:

$$\langle x, y, z \rangle = \langle \frac{\sqrt{3}}{2}, \frac{1}{2}, \frac{1}{2} \rangle + t\langle -\frac{1}{2}, \frac{\sqrt{3}}{2}, -\sqrt{3} \rangle$$