Quiz #3; Wed, 2/10/2016

Math 53 with Prof. Stankova

Section 107; MWF10-11

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Problem. Two lines L_1, L_2 given as follows:

$$L_1 : \quad \frac{x-1}{2} = y - 1 = z - 2$$

$$L_2 : \quad x - 1 = \frac{y-1}{2} = z - 2$$

(a) L_1 and L_2 intersect at a point P. Find P. (5 points)

(b) Let $Q = (5, 3, 4)$ be a point on the line L_1, and let R be a point on line L_2 such that $\overline{QR} \perp \overline{PR}$. Find the coordinates for this point R. (10 points)

Solution. (a) For both L_1 and L_2, the "initial starting point" is $(1, 1, 2)$, so $P = (1, 1, 2)$.

(b) The vector $\overrightarrow{PR}$ can be acquired by projecting the vector $\overrightarrow{PQ}$ onto the direction vector $\mathbf{u} = \langle 1, 2, 1 \rangle$ of the line L_2. That is, $\overrightarrow{PR} = \text{Proj}_{\mathbf{u}} \overrightarrow{PQ} = \dfrac{\langle 1, 2, 1 \rangle \cdot \langle 4, 2, 2 \rangle}{1^2 + 2^2 + 1^2} \mathbf{u} = \dfrac{5}{3} \mathbf{u}$. Thus, the point R, considered as the vector $\overrightarrow{OR} = \overrightarrow{OP} + \overrightarrow{PR}$ is $\langle 1, 1, 2 \rangle + \langle 5/3, 10/3, 5/3 \rangle = \langle 8/3, 13/3, 11/3 \rangle$.