Quiz #1; Wed, 1/27/2016

Math 53 with Prof. Stankova

Section 107; MWF10-11

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Student Name: ______________________________

Problem. (a) (10 points) Sketch the parametric curve C on the x, y–plane defined by

$$x = t + \frac{1}{t}, \quad y = t - \frac{1}{t} \qquad t \neq 0$$

(You need *not* indicate the directionality of the curve, i.e. how $(x(t), y(t))$ travels on the curve C as t is increasing).

(b) (5 points) Find $\frac{dx}{dy}$ (Caution: NOT $\frac{dy}{dx}$) at $t = 1$. (Hint: you may not need to do any computation if you have done part (a)).

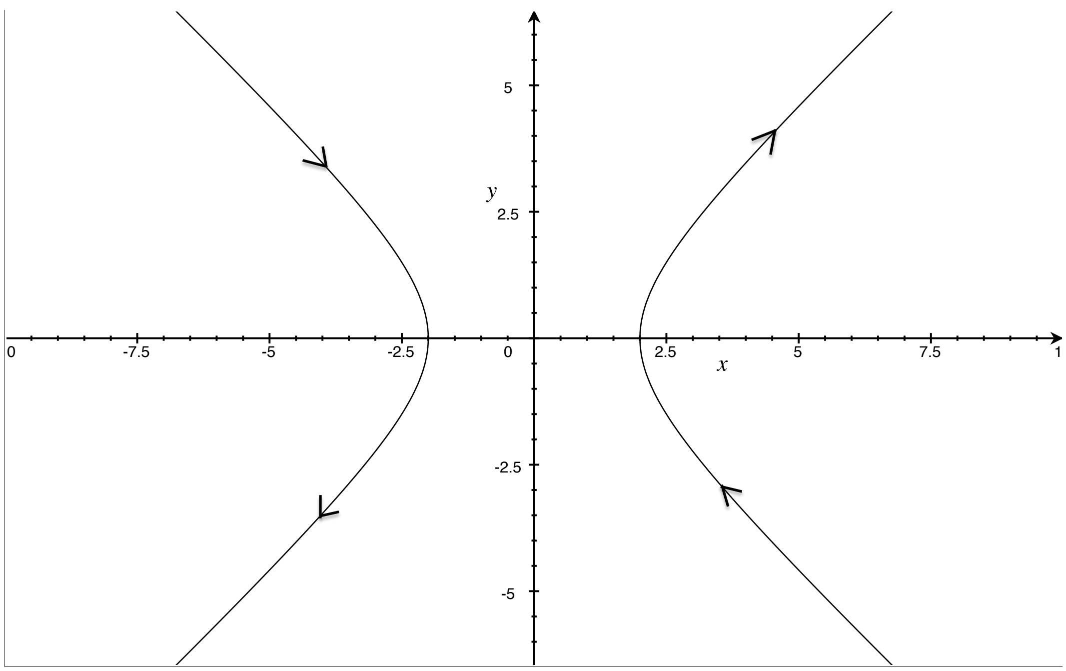
Solution. (a) Note that

$$\begin{aligned} x^2 - y^2 &= (t + \tfrac{1}{t})^2 - (t - \tfrac{1}{t})^2 \\ &= (t^2 + 2 + \tfrac{1}{t^2}) - (t^2 - 2 + \tfrac{1}{t^2}) \\ &= 4 \end{aligned}$$

Thus, the implicit equation for the parametric curve is $\frac{x^2}{2^2} - \frac{y^2}{2^2} = 1$. For $t > 0$, we obtain the right half of the hyperbola since x is always positive then, and likewise for $t < 0$ we get the left half to they hyperbola.

By the following or likewise reasoning, we can determine the directionality of the curve.

If $t \to +\infty$, then $x \to +\infty$ and $y \to +\infty$
If $t \to 0^+$, then $x \to +\infty$ and $y \to -\infty$
If $t \to 0^-$, then $x \to -\infty$ and $y \to +\infty$
If $t \to -\infty$, then $x \to -\infty$ and $y \to -\infty$



(b) At $t = 1$, the point on the curve is $(2, 0)$, at which the tangent line is the vertical line. Hence, $dx/dy = 0$.

Alternatively, one can compute that $\dfrac{dx}{dy} = \dfrac{dx/dt}{dy/dt} = \dfrac{1 - 1/t^2}{1 + 1/t^2}$, which is 0 at $t = 1$.