Quiz #13; Wed, 4/27/2016

Math 53 with Prof. Stankova

Section 107/110; MWF10-11

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Problem. Let $\mathbf{F} := \langle ye^{xy}\sin x + e^{xy}\cos x, xe^{xy}\sin x\rangle$ be a vector field on $\mathbb{R}^2$, and let C be a path from $(1, 0)$ to $(0, 1)$ along the circle of radius 1. Find $\int_C \mathbf{F} \cdot d\mathbf{r}$. (Hint: is $\mathbf{F}$ conservative?)

Solution. One can check that the vector field $\mathbf{F}$ is closed and defined on $\mathbb{R}^2$ which is open and simply-connected. Hence, $\mathbf{F}$ is conservative. Alternatively, we can skip the above step and try finding f such that $\nabla f = \mathbf{F}$, in which case we have $f = e^{xy} \sin x + K$ for any constant K. This also shows that $\mathbf{F}$ is conservative. Now, by FTLI, we have that the line integral is $f(0,1) - f(1,0) = \sin 1 - 0 = \sin 1$.