The approximate Euler method for Lévy driven stochastic differential equations

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Abstract

This talk will discuss the numerical approximation of the expected value $\mathbb{E}(g(X_t))$, where g is a suitable test function and X is the solution of a stochastic differential equation driven by a Lévy process Y. In particular we consider an Euler scheme or an "approximate" Euler scheme with stepsize $1/n$, giving rise to a variable X_t^n which one can simulate, and we study the error $\delta_n(g) = \mathbb{E}(g(X_t^n)) - \mathbb{E}(g(X_t))$.

For a genuine Euler scheme we typically get that $\delta_n(g)$ is of order $1/n$, and we even have an expansion of this error in successive powers of $1/n$, under some integrability condition on the driving process and appropriate smoothness of the coefficient of the equation and of the test function g.

For an approximate Euler scheme, which is when we replace the increments of X by random variables we can simulate that are close enough to the desired increment, the order of magnitude of $\delta_n(g)$ is the supremum of the reciprocal of the number of times we repeat the Euler scheme and a kind of "distance" between the increments of the Lévy process Y and the actual simulated random variable. In this situation, a second order expansion is also available.

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