Modular Arithmetic

JV Practice 7/19/20

Anish Sevekari

Warmup

1. Find the units place of
   (a) 11^{2020}
   (b) 7^{2020}
   (c) 147^{2020}
2. Is $31^{57} - 43^{61}$ a multiple of 11?
3. Explain why divisibility rule of 4, that is, a number is divisible by 4 if and only if the number formed by its last 2 digits is divisible by 4.

Basic Properties and Definitions

We say that a is *congruent* to b *modulo* n, written as

$$a \equiv b \pmod{n}$$

if a and b leave the same remainder after dividing by n. This is equivalent to saying that $n \mid a - b$ (n divides $a - b$).

If $a \equiv b \pmod{n}$ and $c \equiv d \pmod{n}$, then

$$a + c \equiv b + d \pmod{n}$$
$$a - c \equiv b - d \pmod{n}$$
$$a \times c \equiv b \times d \pmod{n}$$

Problems

1. Find the remainder when 555 is divided by 13 (Using Modular Arithmetic!)
2. Find the remainder when 555^2 is divided by 13.
3. Find the remainder when 7^{7^7} is divided by 10.
4. Divisibility test for 3: A natural number written as $\overline{a_n a_{n-1} \dots a_1 a_0}$ in base 10 is divisible by 3 if and only if sum of its digits, that is $a_0 + a_1 + \cdots + a_n$ is divisible by 3.
5. Divisibility test for 11: A natural number written as $\overline{a_n a_{n-1} \dots a_1 a_0}$ in base 10 is divisible by 11 if and only if $a_0 - a_1 + a_2 - \cdots + (-1)^n a_n$ is divisible by 11.

6. Find x such that $2x \equiv 23 \pmod{39}$.
7. Find x such that $3x \equiv 22 \pmod{37}$.
8. Is there a x such that $6x \equiv 22 \pmod{39}$.
9. Find the remainder when $1 \cdot 3 \cdot \ldots \cdot 2019 - 2 \cdot 4 \cdot \ldots \cdot 2020$ is divided by 2021.