Logistics

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Induction

The principle of mathematical induction states that for any mathematical statement $S(n)$, where $n$ is a natural number, if

1. $S(1)$ is true and
2. for every $k \geq 1$, $S(k)$ implies $S(k+1)$

then $S(n)$ is true for every $n \geq 1$.

Examples

1. $S(n) = \text{"the } n\text{-th domino falls"}$
2. $S(n) = \left(1 + 2 + \ldots + n = \frac{n(n+1)}{2}\right)$
3. $S(n) = \text{"}7^n - 4\text{" is a multiple of 3"}
Using induction

(1) Prove that, for every natural number \( n \),

\[ 1 + 2 + \ldots + n = \frac{n(n+1)}{2} \]

**Base case:** \( (n=1) \)

\[ 1 = \frac{1 \cdot 2}{2} \] so the base case holds.

**Induction step:** Suppose that \[ 1 + 2 + \ldots + b = \frac{b(b+1)}{2} \] for some \( b \).

We want to show that \[ 1 + 2 + \ldots + b + (b+1) = \frac{(b+1)(b+2)}{2} \]

This follows from algebraic manipulations:

\[
\begin{align*}
1 + 2 + \ldots + b + (b+1) &= \frac{b(b+1) + (b+1)}{2} \\
&= \frac{(b+1)(b+2)}{2} \\
&= \frac{(b+1)(b+2)}{2}
\end{align*}
\]

(2) Prove that, for every integer \( m \geq 0 \), \( 7^{m} - 4^{m} \) is divisible by 3.

**Base case:** \( (m=0) \)

\[ 7^{0} - 4^{0} = 1 - 1 = 0 \] which is indeed a multiple of 3.

**Induction step:** Suppose that \( 7^{b} - 4^{b} \) is a multiple of 3 for some \( b \geq 0 \). We want to show that \( 7^{b+1} - 4^{b+1} \) is also a multiple of 3. Again, this follows from algebraic
manipulations:

\[ 7^{b+1} - 4^{b+1} = 7 \cdot 7^b - 4 \cdot 4^b \]

\[ = (3+4) \cdot 7^b - 4 \cdot 4^b \]

\[ = 3 \cdot 7^b + 4 \cdot (7^b - 4^b) \]

multiple of 3, by

the induction hypothesis.

We have thus written \( 7^{b+1} - 4^{b+1} \) as a sum of multiples

of 3. We deduce that \( 7^{b+1} - 4^{b+1} \) is itself a multiple of 3.