Question 1. Several people stated that
\[ f(x) \in f(S) \implies x \in S. \]
This is false. If you assume that \( f(x) \in f(S) \) then you may only deduce, using the definition of the image of the set \( S \) under the function \( f \), that there exists some \( y \in S \) such that \( f(x) = f(y) \). You cannot then deduce that \( x = y \), and in particular it is not necessarily the case that \( x \) belongs to \( S \).

Question 3. Some people were not sufficiently careful when defining their explicit bijection \( f : S \to \mathbb{Z} \). For example, they some people defined \( f \) as
\[
  f(x) = \begin{cases} 
    -\frac{\sqrt{x}}{2} & \text{if } \sqrt{x} \text{ is even or} \\
    \frac{\sqrt{x} + 1}{2} & \text{if } \sqrt{x} \text{ is odd.}
  \end{cases}
\]
This is almost correct, but not quite, since it is not a surjection. Indeed, the image of \( \{0^2, 2^2, 4^2, 6^2, \ldots \} \) will be \( \{-1, -2, -3, \ldots \} \) whilst the image of \( \{1^2, 3^2, 5^2, \ldots \} \) will be \( \{1, 2, 3, \ldots \} \) but 0 will never be attained via \( f \).

The moral of the story is to be particularly careful when it comes to edge cases (such as 0 in this problem).

Question 9. A recurring mistake was to leave the definite sum
\[
\sum_{i=1}^{11} i
\]
unevaluated, i.e. keeping it in that form instead of evaluating it to 66. Your final answer should be in a reasonably simplified form, and just as you would not leave the definite integral
\[
\int_{0}^{11} x \, dx
\]
unevaluated in a calculus problem, you should not leave the definite sum
\[ \sum_{i=1}^{11} i \]
unevaluated in a combinatorics problem.

**Question 10.** (What follows did not impact folks’ grades, but may be interesting nonetheless)

There were many ways to write down the answer for this question, and it is an important skill to be able to write down your answer in the simplest form possible. Here is therefore a quick note showing that certain sums can be re-written in simpler ways by changing variables (as you would for integrals): by performing the change of variable \( i \rightarrow j = n - i \) we see that
\[ \sum_{i=1}^{n} (n - i) = \sum_{j=0}^{n-1} j. \]

You would then be able to use the well-known identity \( \sum_{j=1}^{n} j = \frac{n(n+1)}{2} \) in order to evaluate the original sum. (PS: If you are bored, prove this identity by counting in two ways).