Common mistakes: HW 2

September 19, 2019

Question 2: Proving set inclusion 2. (Part (a) in particular)

Some people attempted to give a proof that for arbitrary sets $A$ and $B$,

$$A \times B^c \not\subseteq A^c \times B^c.$$ 

This cannot work, since there are examples of sets $A$ and $B$ for which $A \times B^c$ is not a subset of $A^c \times B^c$ (you were tasked with finding such an example), but there are also examples of sets $A$ and $B$ for which $A \times B^c$ is a subset of $A^c \times B^c$. For example, if $A = \{1\}$, $B = \{1\}$ and the universal set is $\mathcal{U} = \{1\}$ then $A \times B^c = A^c \times B^c$.

Question 9: Indexed sets 1. (Especially the set involving the indexed union) A lot of people tried to prove that $(0, \infty) \subseteq \bigcup_{r>1} B_r$ by saying that for every $x > 0$, $x \in \left(\frac{1}{x+1}, x+1\right)$. This only works for some $x$, because it is not true for every strictly positive $x$ that $x > \frac{1}{x+1}$. An example of when this fails is $x = \frac{1}{2}$, since then $\frac{1}{\frac{3}{2}} = \frac{2}{3}$ and so indeed $x \not> \frac{1}{x+1}$.

Question 10: Indexed sets 2. Several people argued along the lines of 

"$\mathcal{P}(\mathbb{N})$ is not a subset of $\bigcup_{n=1}^{\infty} \mathcal{P}([n])$ because $\mathbb{N}$ is an infinite set which belongs to $\mathcal{P}(\mathbb{N})$ and which cannot belong to the union, since all elements in the union are finite sets". This is the right idea, but not a rigorous proof.

A rigorous proof would have to say that $\mathbb{N}$ cannot be an element of any of the power sets in the union. To show that $\mathbb{N}$ is not an element of $\mathcal{P}([n])$ means showing that $\mathbb{N}$ is not a subset of $[n]$, which means finding an element which is in $\mathbb{N}$ but not in $[n]$. That element can be taken to be $n$.

Note that this paragraph here indicates more how to find the proof than how to write it – please take a look at Irina’s solution for an example of how to write the proof.