Math 122   **Fall 2008**

Handout 3(a): Calculating Riemann Sums with a TI-84

The purpose of this handout is to show you how to use the summation capabilities of a TI-84 graphing calculator to approximate areas under curves. The sums used to approximate areas under curves are called *Riemann sums*.

Riemann sums can be left-hand Riemann sums or right-hand Riemann sums depending on whether left end-points or right end-points are used to determine the heights of the rectangles.
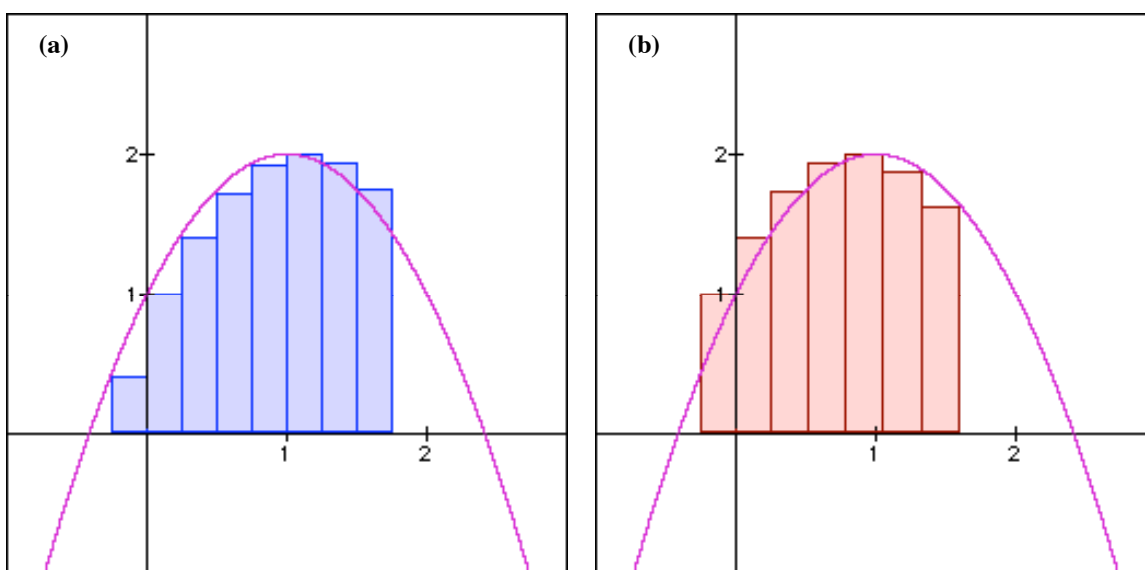

Figure 1: (a) Rectangles representing a left-hand Riemann sum. (b) Rectangles representing a right-hand Riemann sum.

Example

Draw the rectangles that are used to approximate the area under the curve:

$$y = f(x) = 2 \cdot e^{-(x-1)^2}$$

between $a = -0.5$ and $b = 1.5$. Assume that a left-hand Riemann sum with $N = 40$ rectangles.

Solution

The graph of $y = f(x) = 2 \cdot e^{-(x-1)^2}$ is shown on the next page (see Figure 2), together with the rectangles that are used to approximate the area.

Note that since a left-hand Riemann sum is used, the height of each skinny green rectangle is determined by the value of $f(x)$ at the left end-point of each sub-interval.

Note also that the width of each skinny green rectangle is equal to 0.05. This width, often written as Δx, can be calculated from the end-points ($a = -0.5$ and $b = 1.5$) and the number of rectangles used ($N = 40$).
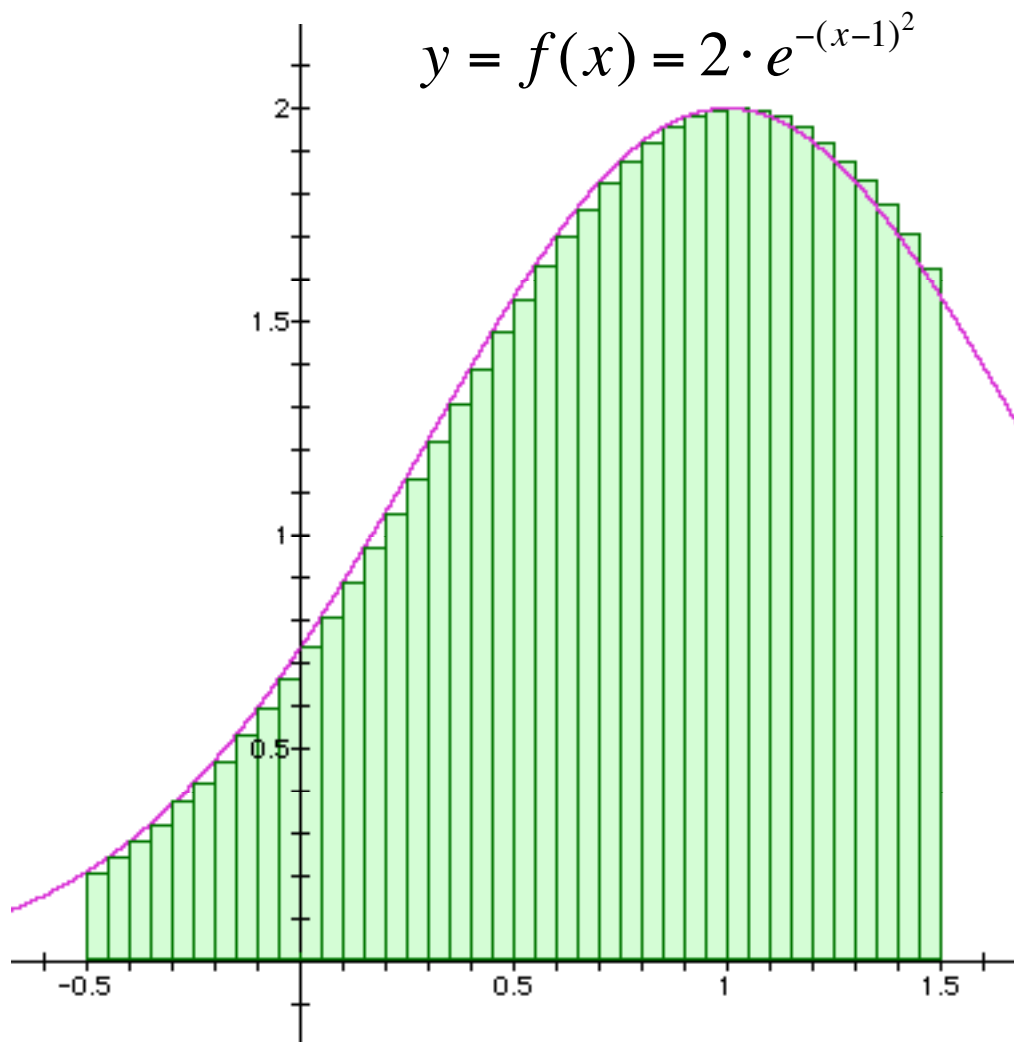
$$\Delta x = \frac{b-a}{N} = \frac{1.5--0.5}{40} = 0.05.$$

$$y = f(x) = 2 \cdot e^{-(x-1)^2}$$

Figure 2: The 40 "skinny green" rectangles used to approximate the area under the curve.

Example

Use summation (sigma or Σ) notation to write sown a symbolic expression that represents the area of the skinny green rectangles in Figure 2 and approximates the area under the

curve $y = f(x) = 2 \cdot e^{-(x-1)^2}$ between $a = -0.5$ and $b = 1.5$. Again, assume a left-hand Riemann sum with $N = 40$ rectangles.

Solution

For our purposes, left-hand Riemann sum will always have the following format when written using summation (or "sigma" notation):

$$\text{Area under curve} \approx \sum_{k=0}^{N-1} f(a + k \cdot \Delta x) \cdot \Delta x.$$

In this expression, $f(a + k \cdot \Delta x)$ represents the height of a rectangle and Δx represents the width of a rectangle. Multiplying these together gives:

$$f(a + k \cdot \Delta x) \cdot \Delta x = (\text{height}) \cdot (\text{width}) = \text{Area of the } k^{th} \text{ skinny green rectangle.}$$

The index of summation, k, keeps track of which skinny green rectangle we are currently finding the area for, and the "sigma" at the beginning of the notation means add all of these areas together. The "$k = 0$" at the bottom of the "sigma" means that the sum should begin with the value of $k = 0$ and the "$N - 1$" at the top of the "sigma" means that the sum should end when the value of k reaches $N - 1$. (In this particular example, $N = 40$ so the sum should end when k reaches $40 - 1 = 39$.)

In this particular example, $y = f(x) = 2 \cdot e^{-(x-1)^2}$, $a = -0.5$, $N = 40$, and $\Delta x = 0.05$. The area under the curve will be approximated by the sum of the areas of the skinny green rectangle:

$$\text{Area under curve} \approx \sum_{k=0}^{39} f(-0.5 + k \cdot 0.05) \cdot 0.05,$$

or, using the formula for $f(x)$:

$$\text{Area under curve} \approx \sum_{k=0}^{39} 2 \cdot e^{-(-0.5 + k \cdot 0.05 - 1)^2} \cdot 0.05.$$

Example

Use the "sum" and "seq" commands on a TI-84 calculator to evaluate the Riemann sum that you wrote down in the previous example.

Solution

The most direct calculator commands that you would use to calculate the numerical value of the left-hand Riemann sum:

$$\text{Area under curve} \approx \sum_{k=0}^{39} 2 \cdot e^{-(-0.5+k\cdot 0.05-1)^2} \cdot 0.05$$

are as follows:

```
sum(seq(2*(e^(-(-0.5+K*0.05-1)^2))*0.05,K,0,39))
```

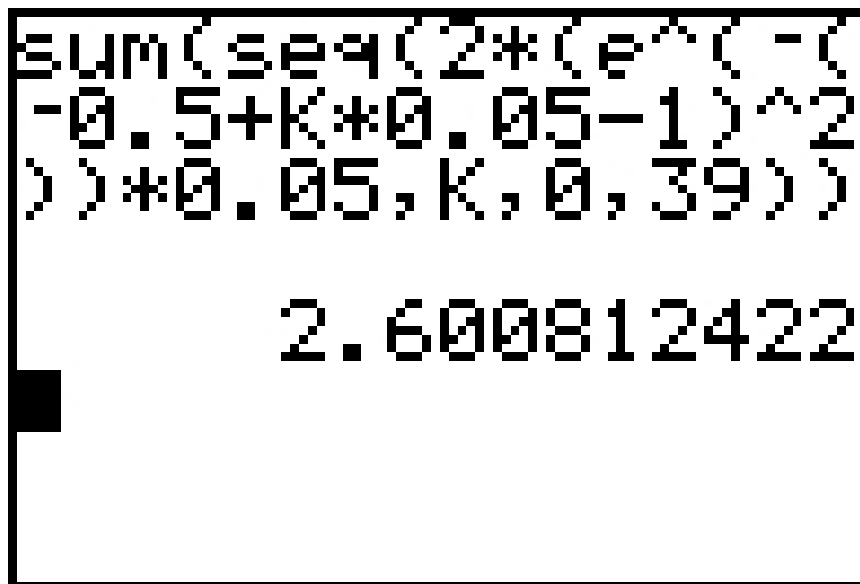The result of executing this command on a TI-84 calculator is shown below.

Figure 3: Result of executing calculator commands on a TI-84.

So, the numerical value of the left-hand Riemann sum is 2.600812422. This represents the combined area of all of the skinny green rectangles shown in Figure 2.

In the rest of this solution we will indicate how the summation ("sigma") notation that mathematicians use to denote a sum and the calculator commands that tell a TI-84 to evaluate a sum are related. We will then indicate how the calculator commands can be accessed on a TI-84 calculator.

Correspondence Between Summation Notation and Calculator Commands

$$\text{Area under curve} \approx \sum_{k=0}^{39} 2 \cdot e^{-(-0.5+k\cdot 0.05-1)^2} \cdot 0.05$$

Last is the value of k where the sum ends

The "sum(seq(" at the start is like the Σ. These tell the calculator to do a sum.

Next is the value of k where the sum begins

```
sum(seq(2*(e^((-0.5+K*0.05-1)^2))*0.05,K,0,39))
```

The formula giving the heights of the rectangles is multiplied by the width, Δx.

The "K" tells the calculator which letter is the index of summation

Immediately after the "sum(seq(" comes the formula for $f(a + k\Delta x)$

$$\text{Area under curve} \approx \sum_{k=0}^{39} 2 \cdot e^{-(-0.5+k\cdot 0.05-1)^2} \cdot 0.05$$

Figure 4: Correspondence between conventional summation notation and TI-84 summation notation.

Steps Involved in Evaluating a Left-Hand Riemann Sum on a TI-84 Calculator

The steps involved in evaluating a left-hand Riemann sum on a TI-84 are written out below. The appearance of the calculator screen and the actual buttons that you have to press at each step of the calculation are shown on the next page.

Step 1: Go to the formula entry screen and enter the formula for $f(x)$ into the "Y1" slot.

Step 2: Store the number of rectangles (N) in calculator memory "N."

Step 3: Store the left end-point (a) in calculator memory "A."

Step 4: Work out the value of Δx ($= \frac{b-a}{N}$) and store this in calculator memory "W."

Step 5: Execute the calculator command:

```
sum(seq(Y1(A+K*W)*W,K,0,N-1))
```

The calculator will evaluate the Riemann sum (although it may take some time if you are using a large number of rectangles) and report its numerical value.
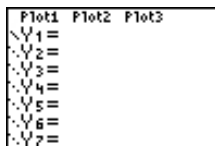
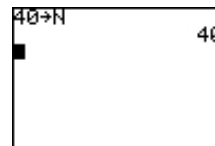**(a)** Press the [Y=] button to enter the formula for $f(x)$
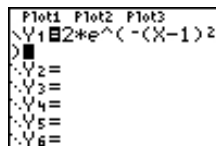
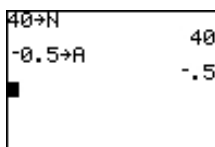**(b)** Enter the formula for $f(x)$ in the Y1 slot

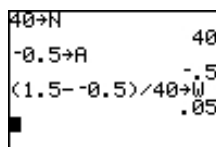**(c)** Press [2nd] "QUIT" to return to the main screen
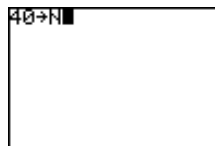
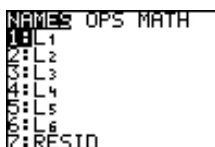**(d)** Store the number of rectangles in memory "N." The little arrow is produced by the "STO" button.

(e) Pressing "ENTER" will store the value in memory "N"

(f) In a similar way, store the left end-point in memory "A"

(g) In a similar way, store the value of Δx in memory "W"

(h) Press [2nd] "LIST" to access the LIST menus

(i) Use the arrow keys to go across to the MATH menu

(j) Use the arrow keys to go down to option 5 "sum("
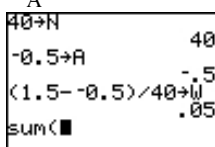
(k) Press the [ENTER] key
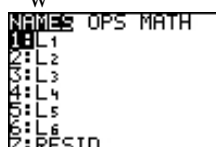
(l) Press [2nd] "LIST" to access the LIST menus
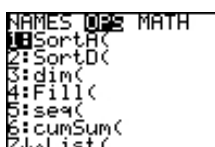
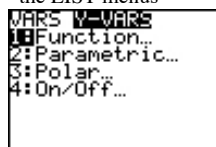**(m)** Use the arrow keys to go across to the OPS menu
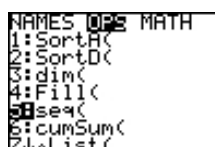
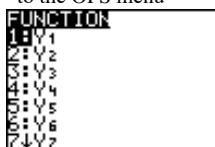**(n)** Use the arrow keys to go down to option 5 "seq("
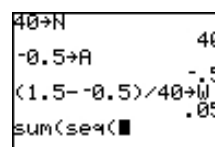
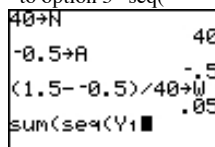**(o)** Press the [ENTER] key

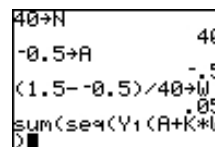**(p)** Press the [VARS] key

(q) Use the arrow keys to go across to the Y-VARS menu
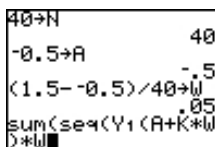
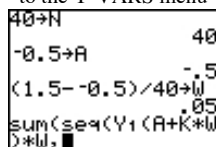**(r)** Press the [ENTER] key

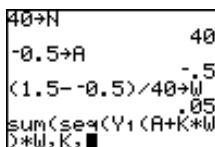**(s)** Press the [ENTER] key

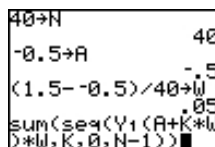**(t)** Finish typing the rest of the formula for $f(a+k\Delta x)$

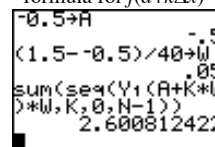**(u)** Multiply $f(a+k\Delta x)$ by "W," the width of the rectangles

(v) Type a comma to indicate the end of formula that gives rectangle areas

(w) Next type "K" (followed by a comma) to identify the index of summation

(x) Enter the values of "K" where the sum begins and ends

(y) Press [ENTER]

Steps Involved in Evaluating a Right-Hand Riemann Sum on a TI-84 Calculator

The steps 1 through 4 are exactly the same as for a left-hand sum. The only difference is that the sum begins with $k = 1$ and ends with $k = N$, so that in Step 5 you would execute the calculator command:

```
sum(seq(Y1(A+K*W)*W,K,1,N))
```