Rigidity and d-dimensional algebraic connectivity of graphs

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Carnegie Mellon University

(based on joint work with Eran Nevo, Yuval Peled, Orit Raz, Michael Krivelevich, Peleg Michaeli)

Rigidity

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- G=(V,E) a graph

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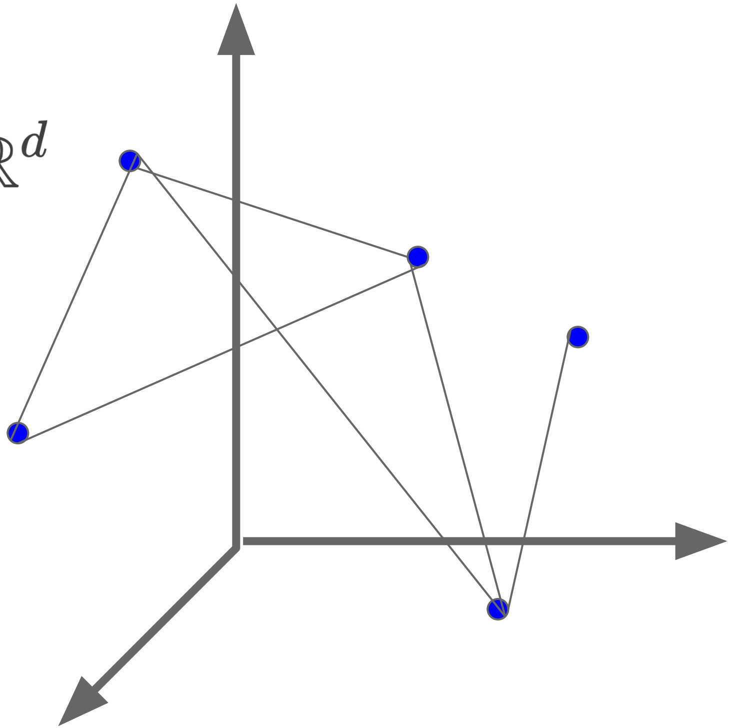

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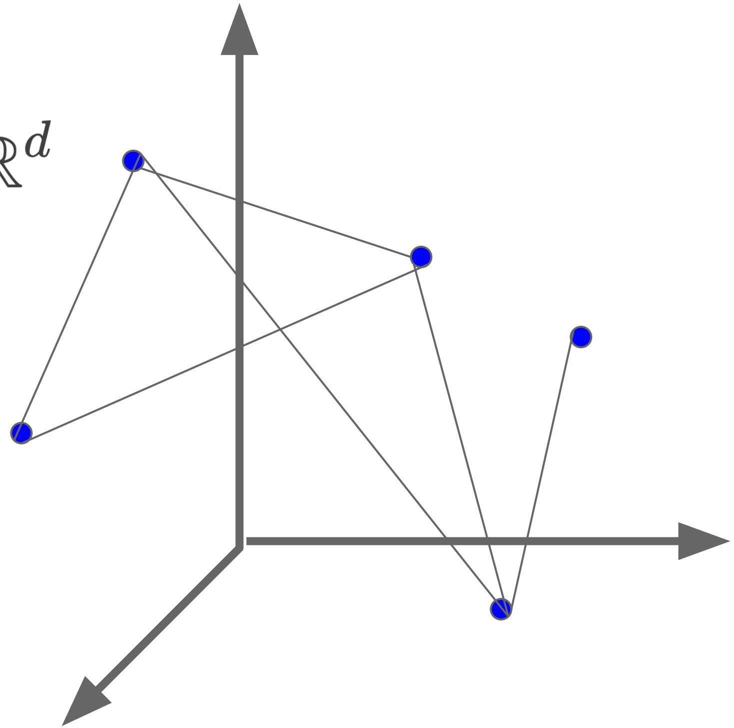

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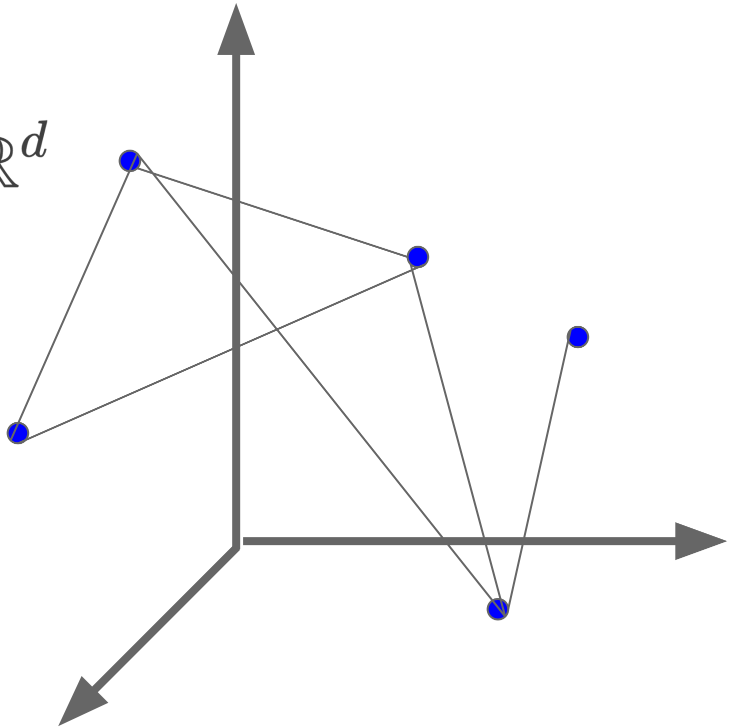

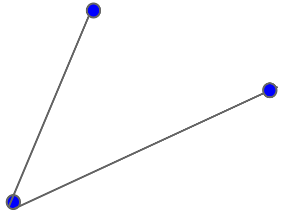

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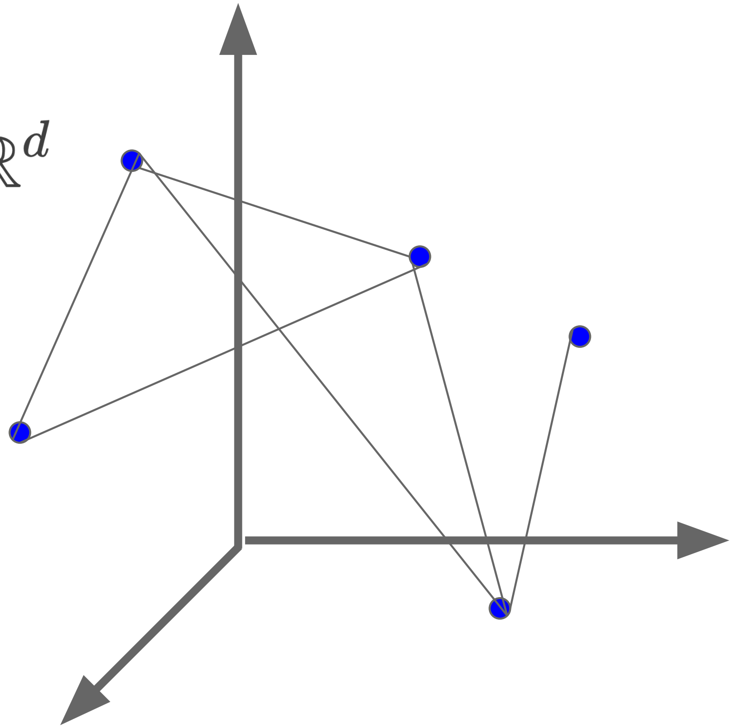

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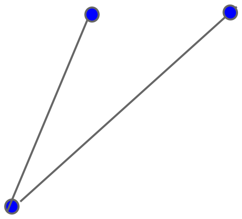

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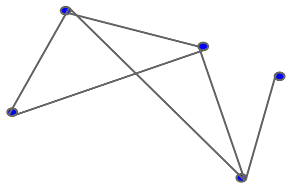

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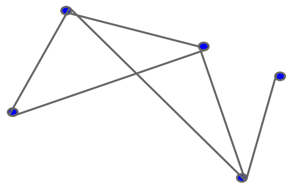

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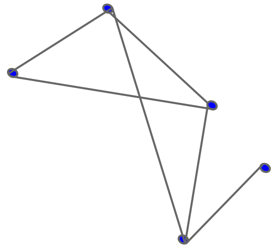

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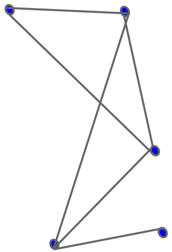

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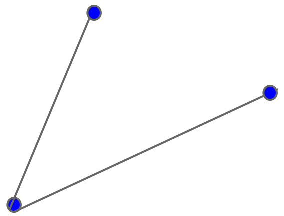
Question: Is the structure **rigid** or **flexible**?

Or: Is there a continuous motion of the vertices that preserves the lengths of all edges, except **translations** and **rotations**?

Some examples



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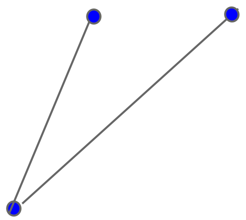

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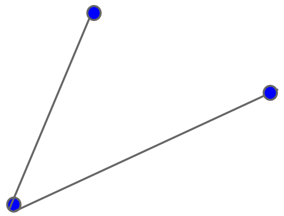

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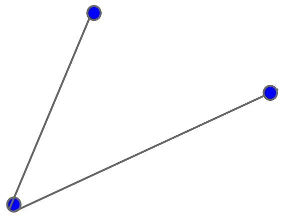

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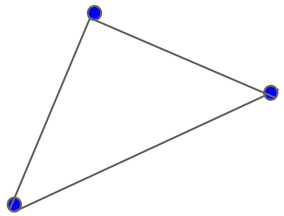

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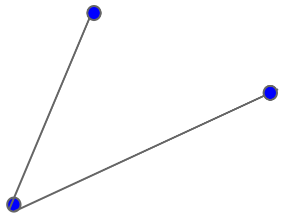

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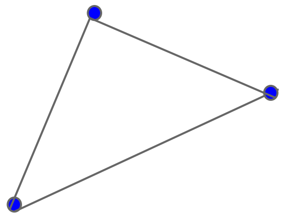

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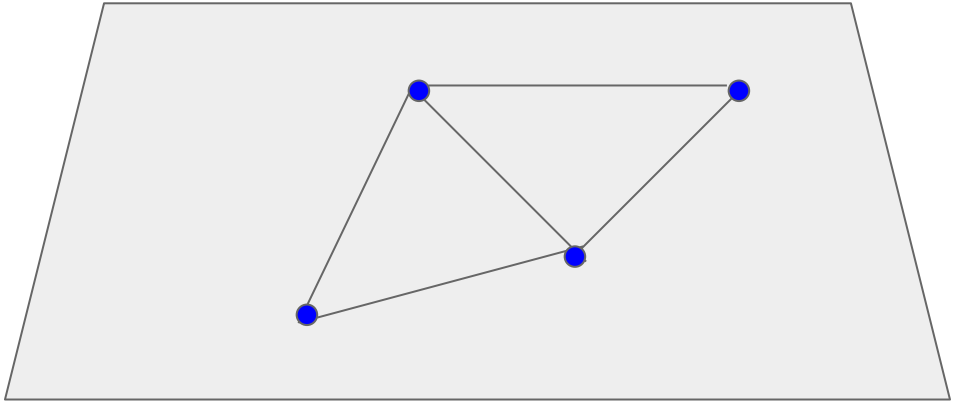

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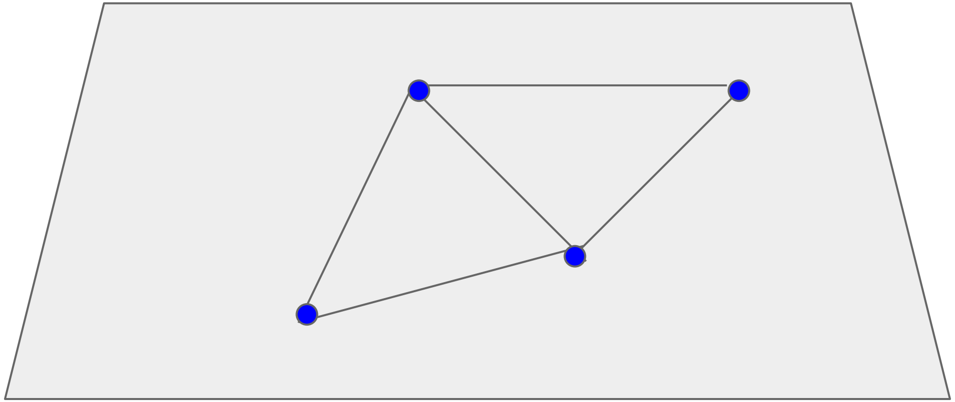

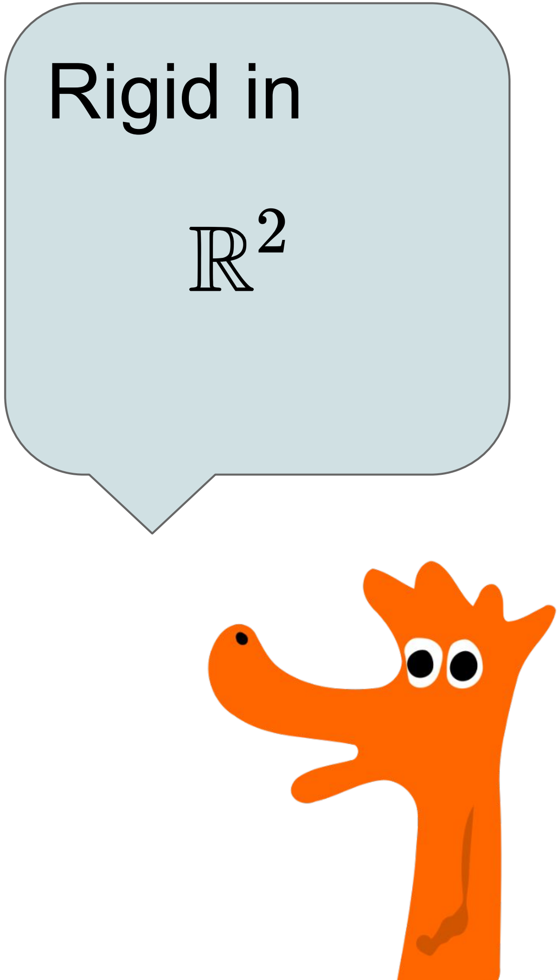

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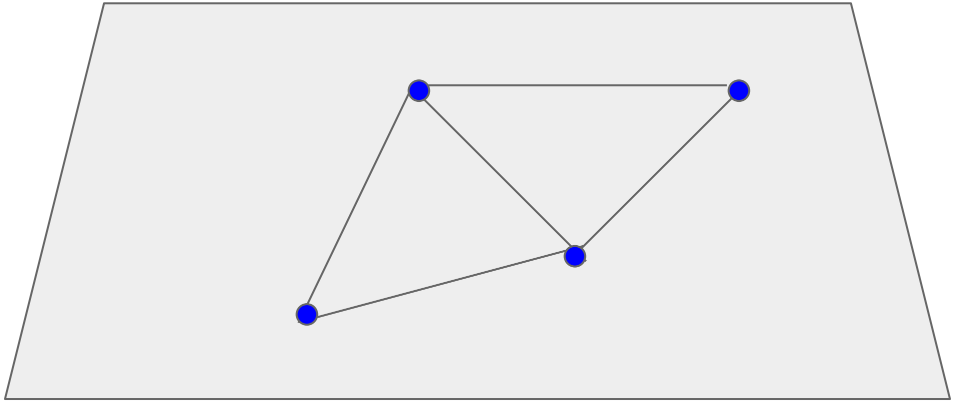

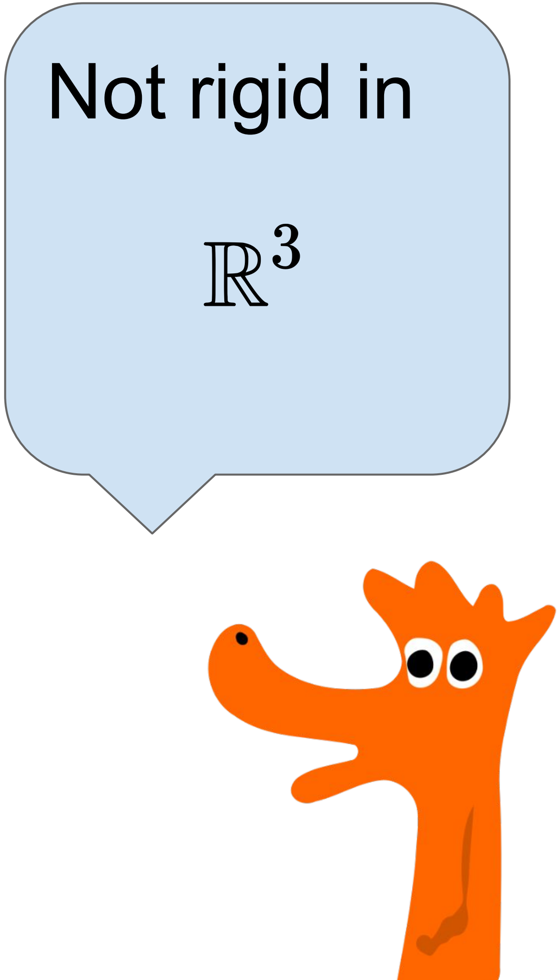

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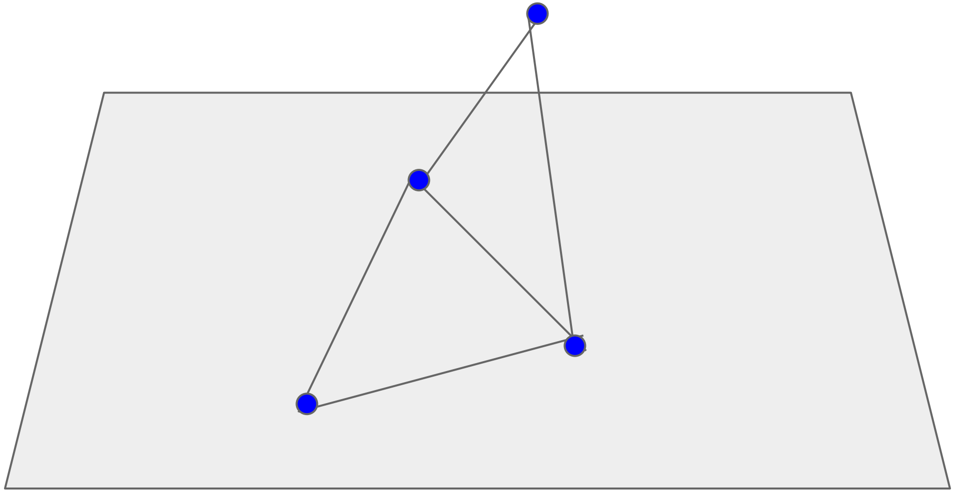

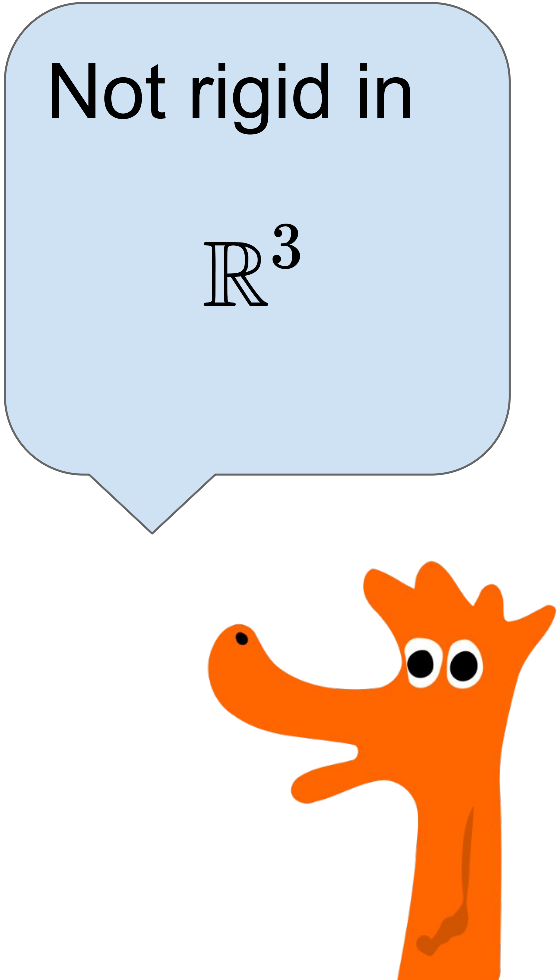

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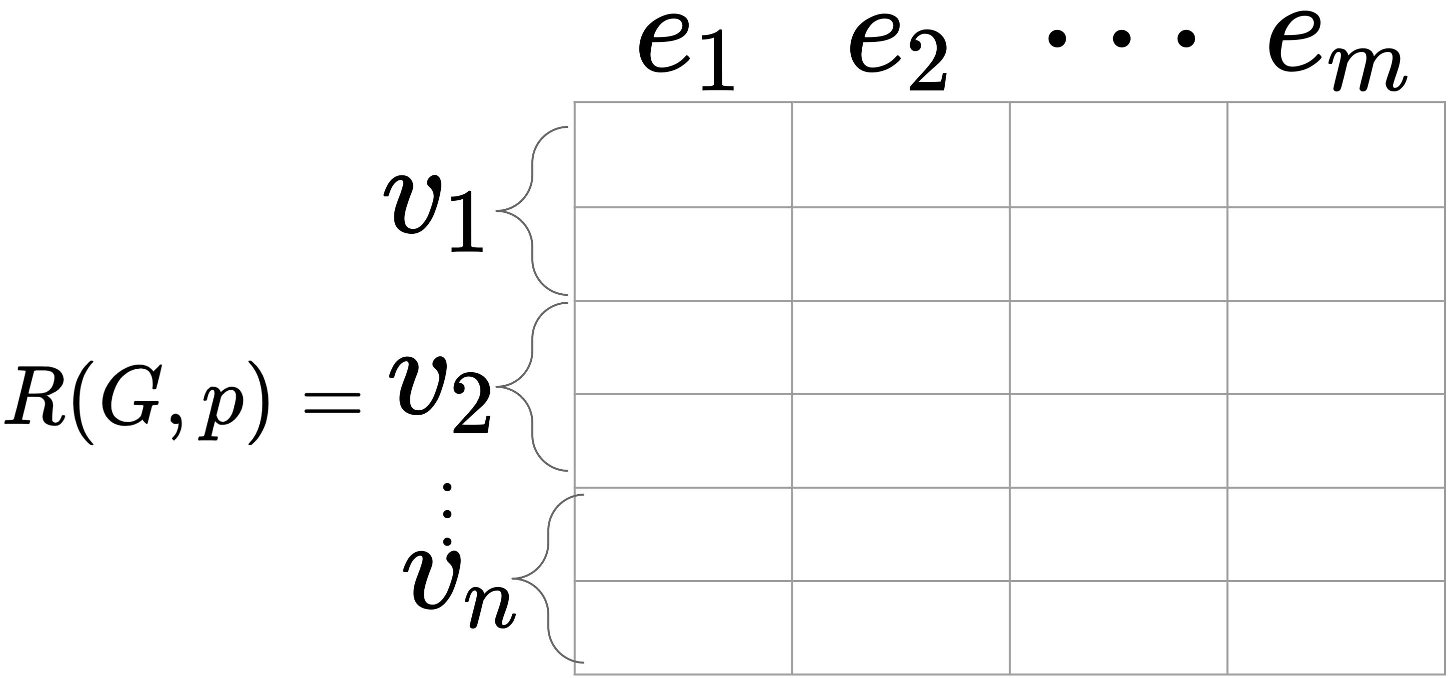

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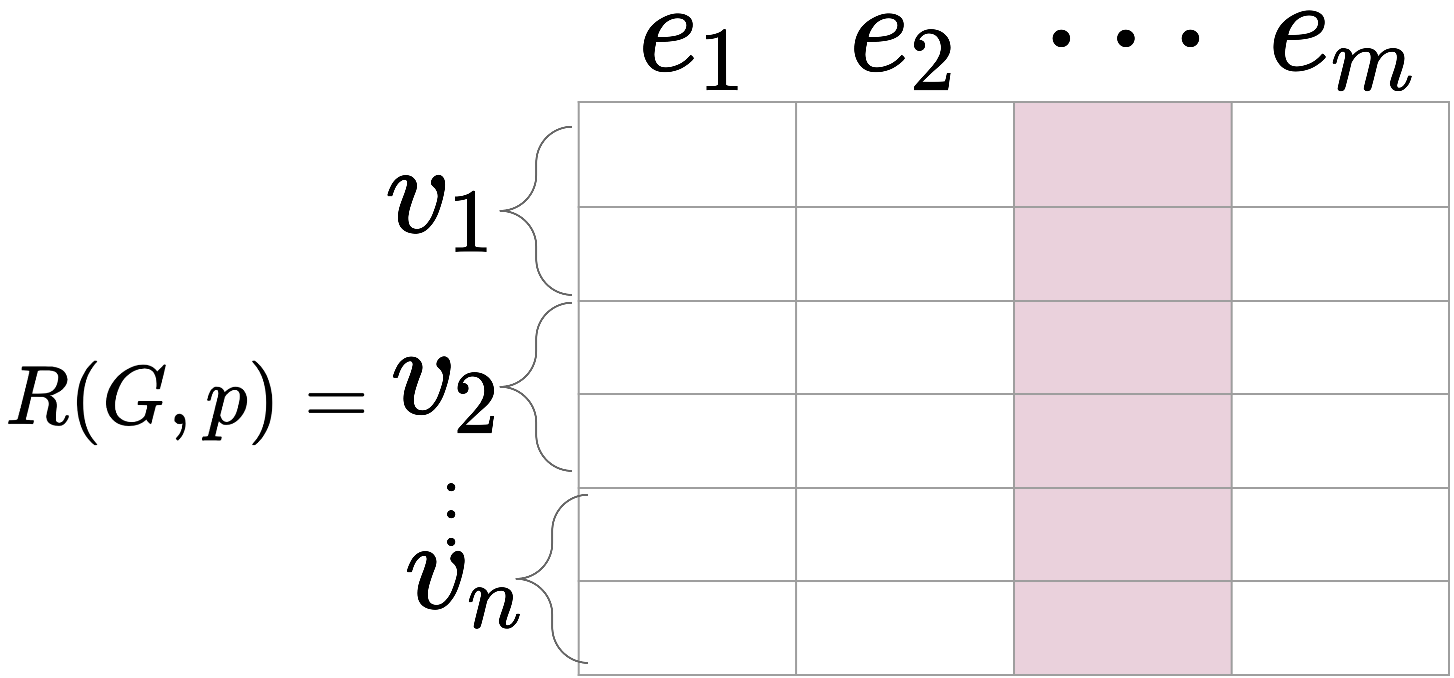

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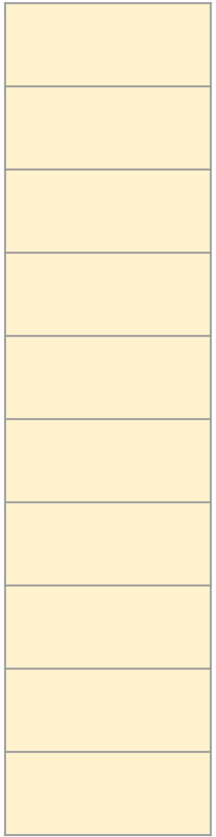
$$e = \{u, v\}$$



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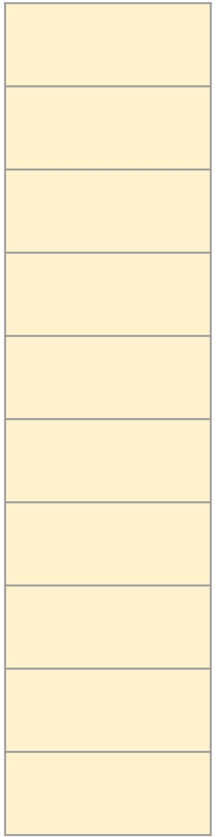

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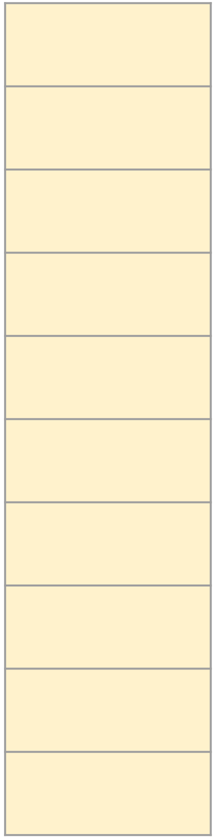

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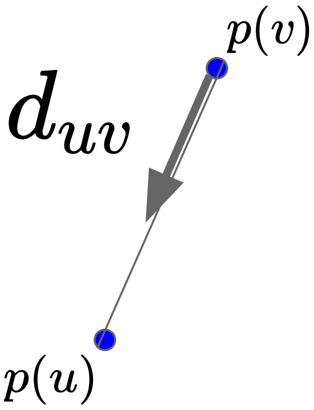

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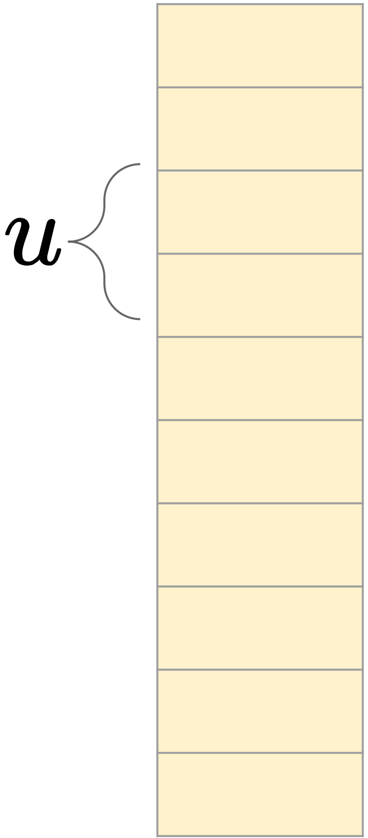

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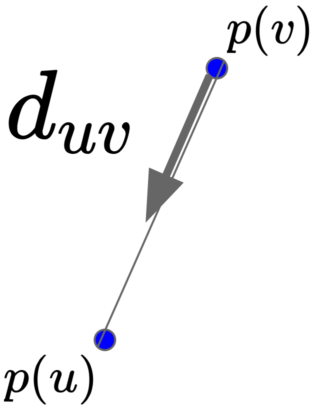

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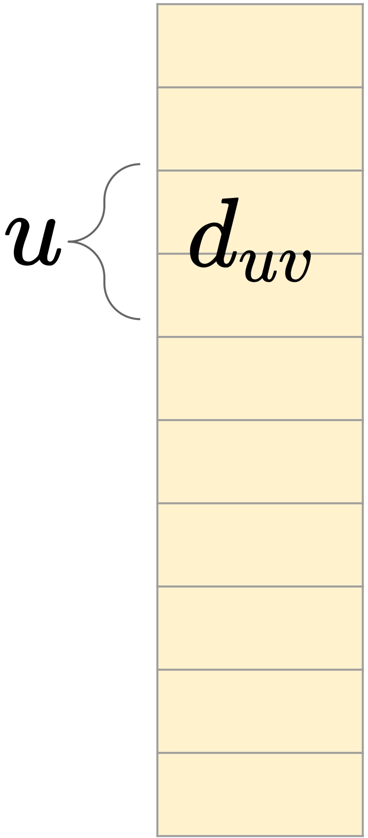

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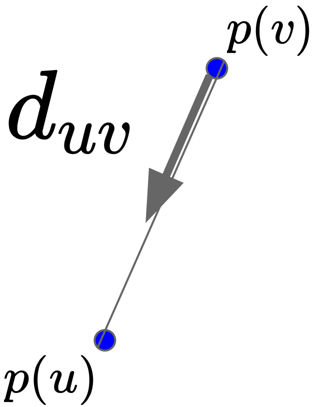

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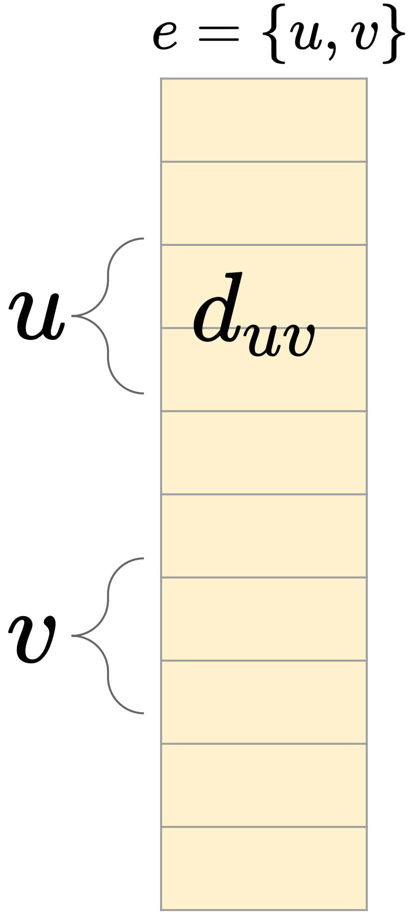

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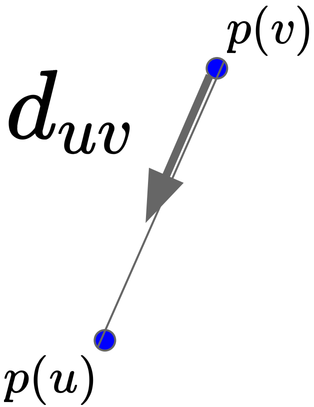

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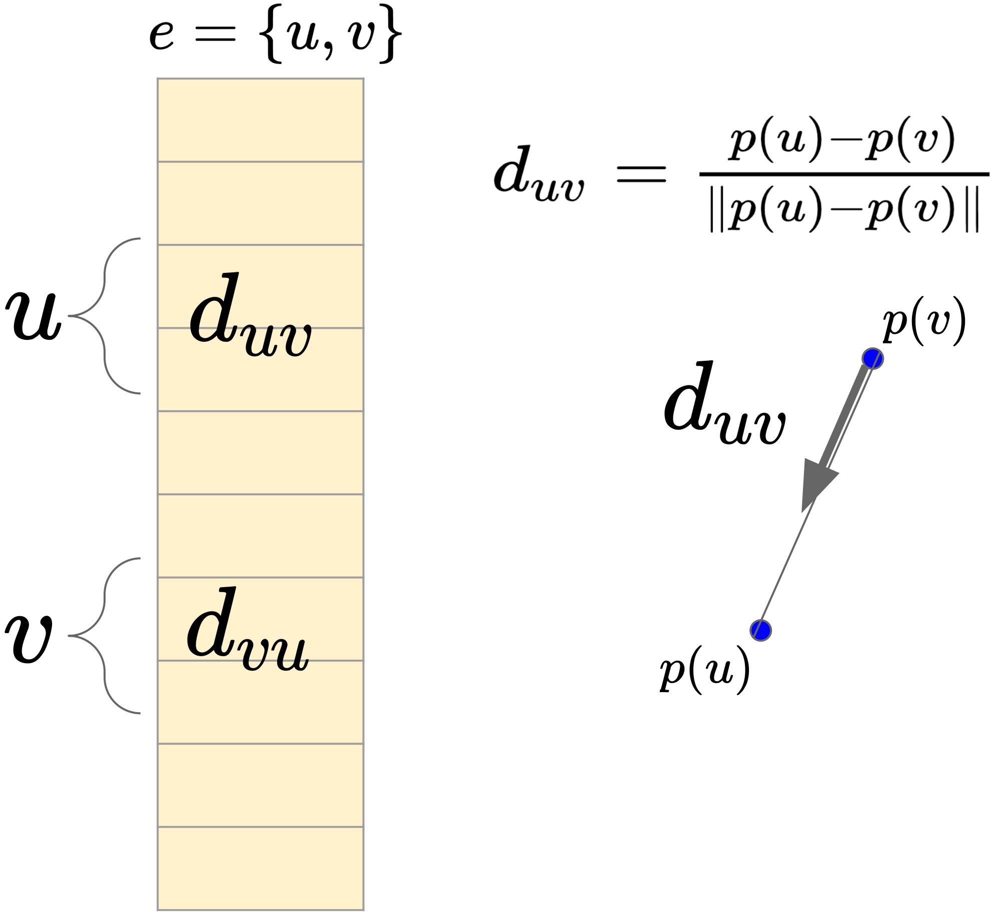

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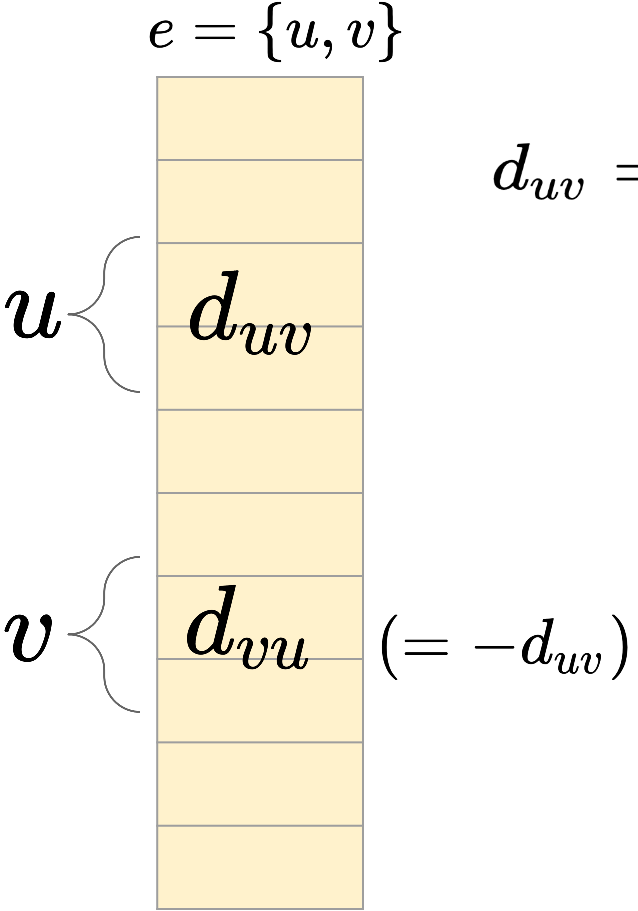

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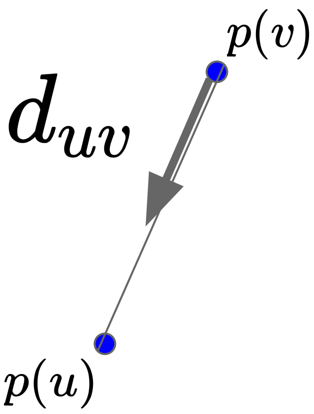

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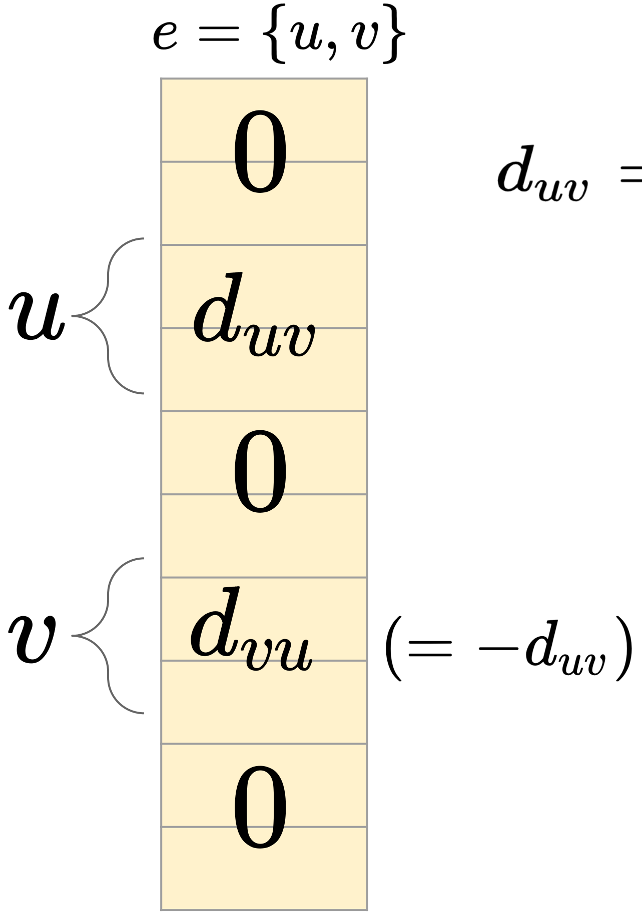

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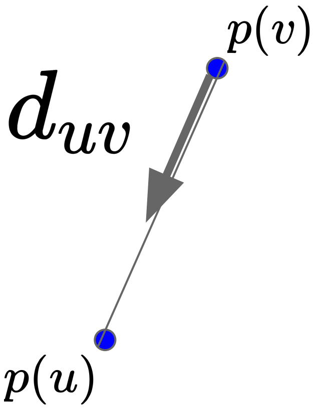

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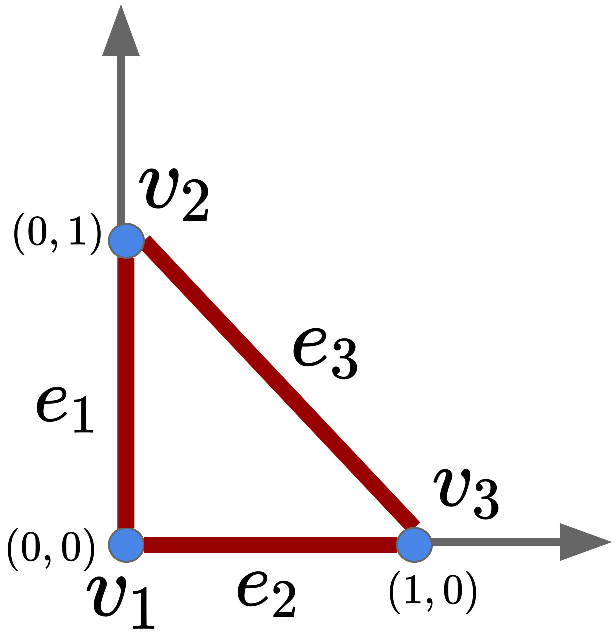

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Example:

$R(G,p) =$

	e_1	e_2	e_3
v_1	0	-1	0
	-1	0	0
v_2	0	0	$-1/\sqrt{2}$
	1	0	$1/\sqrt{2}$
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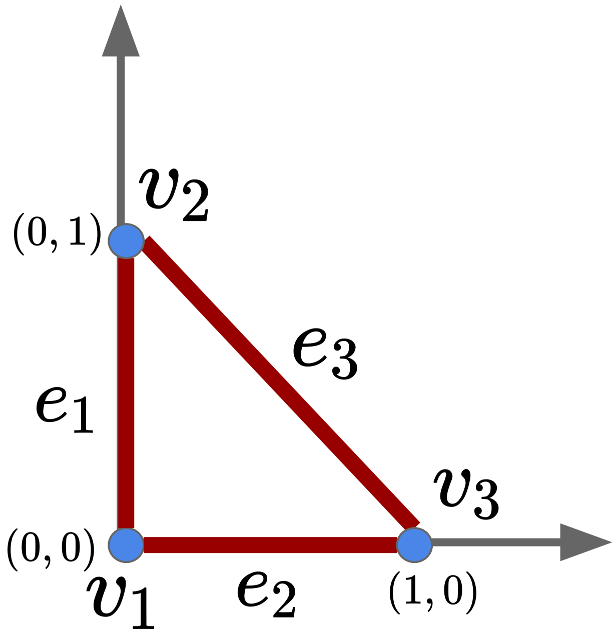

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- $\operatorname{rank}(R(G,p)) \leq dn - \binom{d+1}{2}$

- If $\operatorname{rank}(R(G,p)) = dn - \binom{d+1}{2}$ then (G,p) is rigid.

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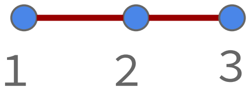

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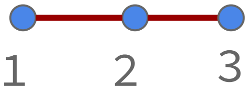

$$R(G,p) = N(G) = \begin{pmatrix} -1 & 0 \\ 1 & -1 \\ 0 & 1 \end{pmatrix}$$

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The Laplacian matrix: $L(G) = N(G)N(G)^T$

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Large algebraic connectivity implies that G is “strongly connected”.

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Spectral gap: $\lambda_{\binom{d+1}{2}+1}(L(G,p))$

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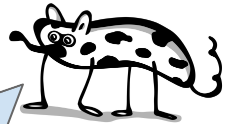

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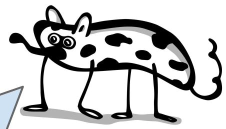

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-If $a_d(G)$ is large enough, then G remains d-rigid (with positive probability) even after removing some of the edges of G uniformly at random.

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Conjecture (L-Nevo-Peled-Raz '22+):

$$a_d(K_n) = \begin{cases} 1 & \text{if} \quad d+1 \leq n \leq 2d, \\ \frac{n}{2d} & \text{if} \quad 2d \leq n. \end{cases}$$

The simplex graph $a_d(K_{d+1}) = 1$

Lower bound:

If p maps the vertices of K_{d+1} into the vertices of a regular simplex in $\mathbb{R}^d$, then the spectrum of $L(K_{d+1}, p)$ is:

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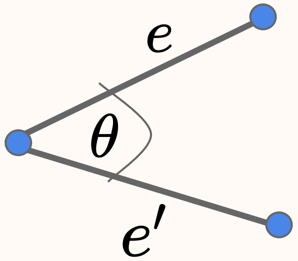

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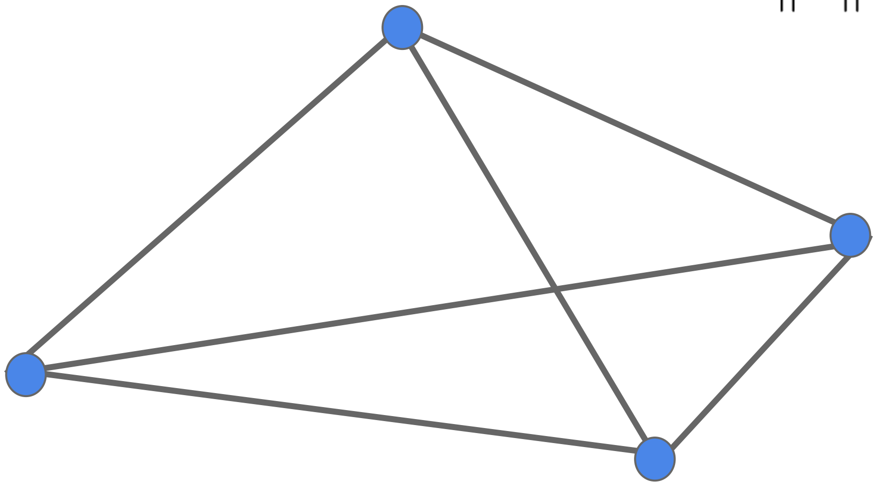

THE SIMPLEX GRAPH

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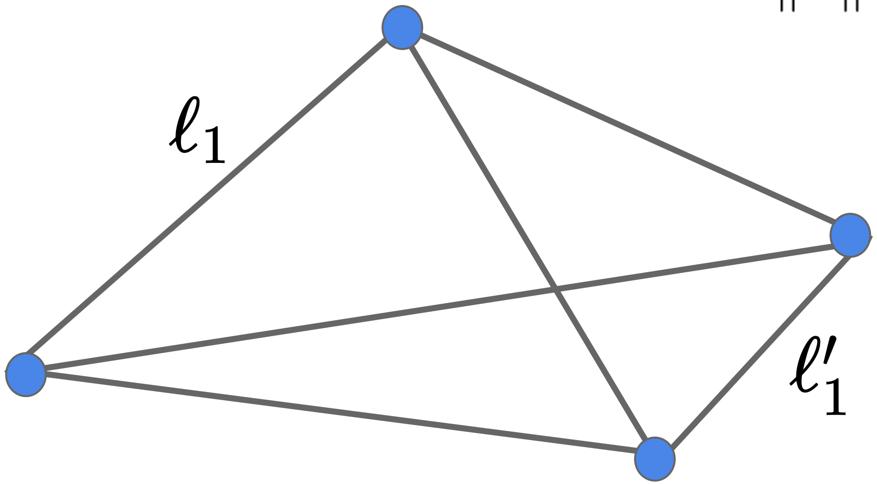
$x =$ 

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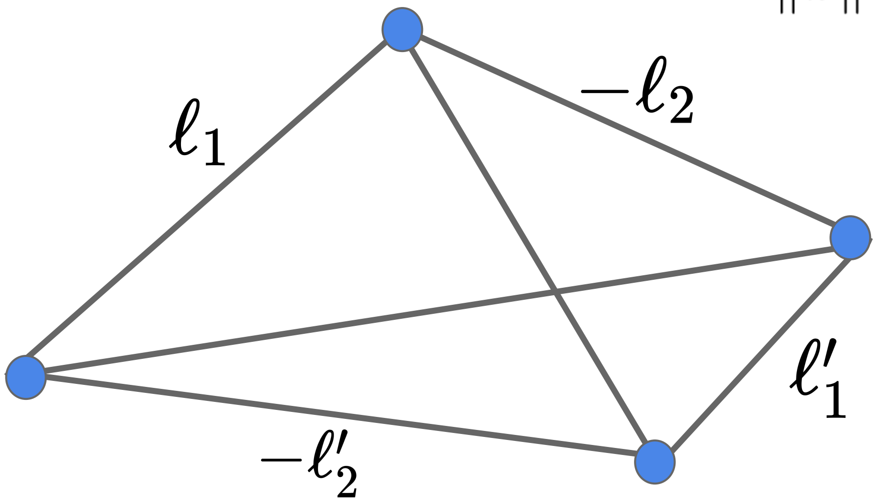

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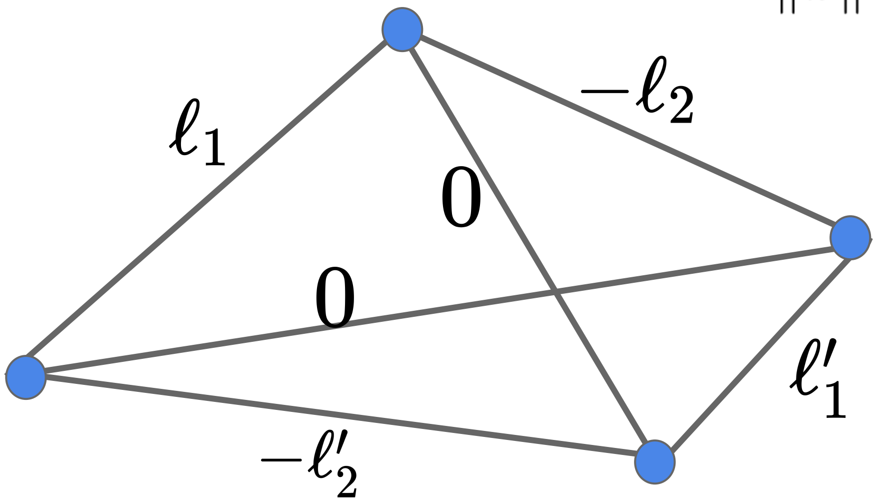
$x =$ 

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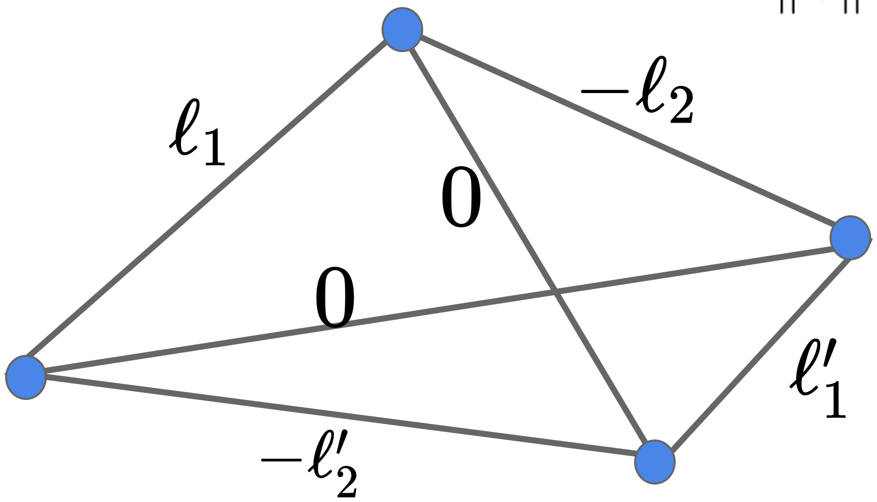
$x =$ 

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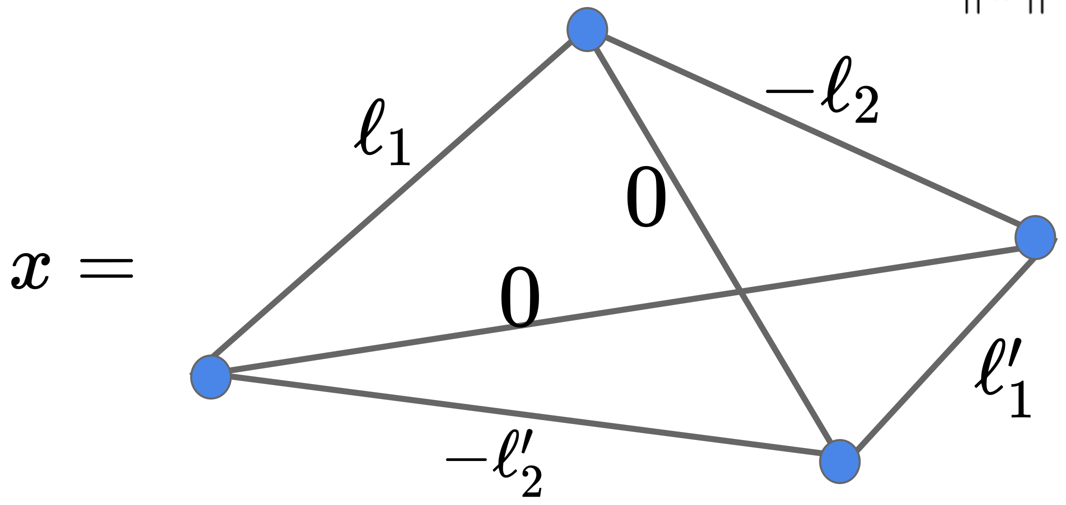

For general d, argue by induction using eigenvalue interlacing

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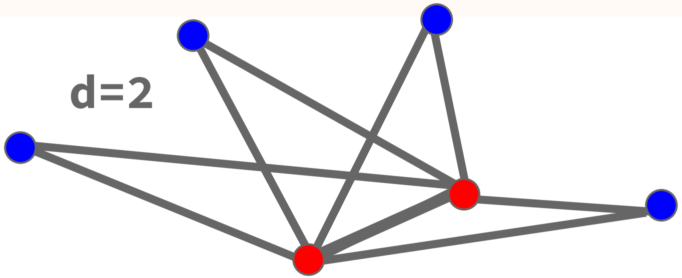

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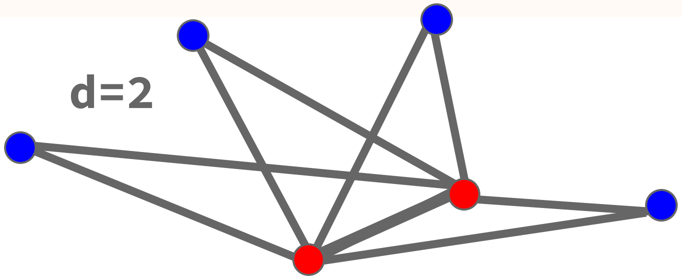

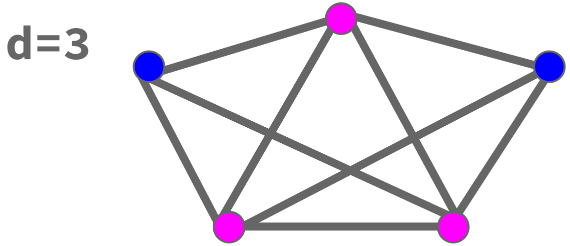

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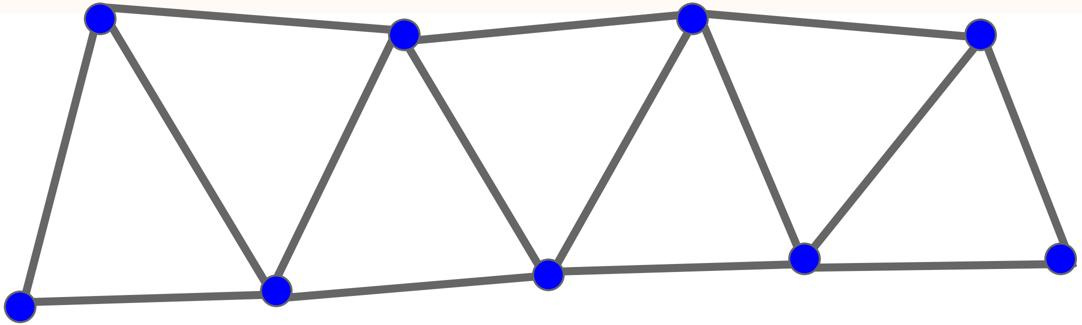

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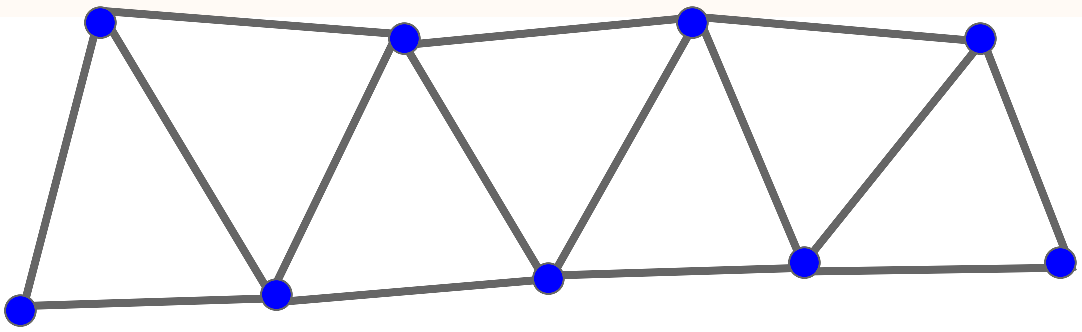

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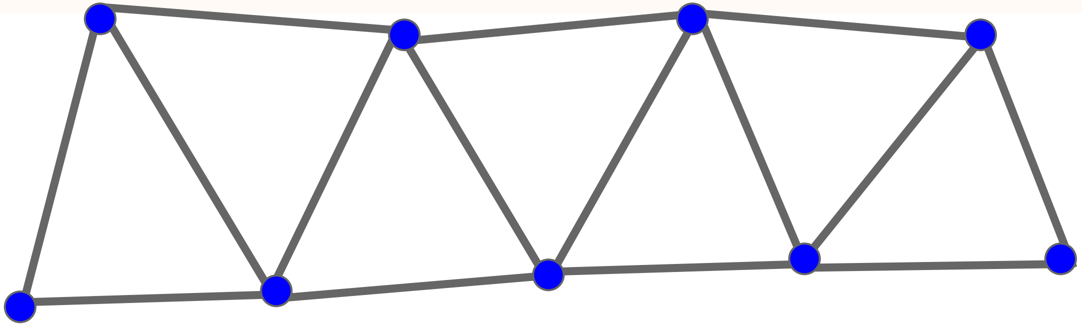

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A family of graphs $\{G_i = (V_i, E_i)\}_{i=1}^{\infty}$ with $\lim_{i\to\infty} |V_i| = \infty$ is a family of **d-rigidity expander graphs** if there is $\epsilon > 0$ such that $a_d(G_i) \geq \epsilon$ for all i.

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What happens for d>1?

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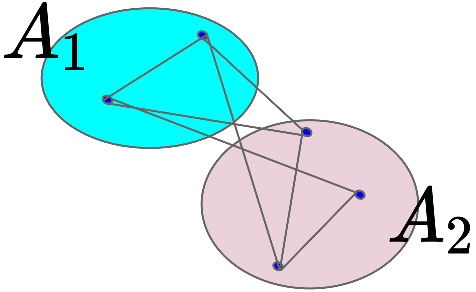

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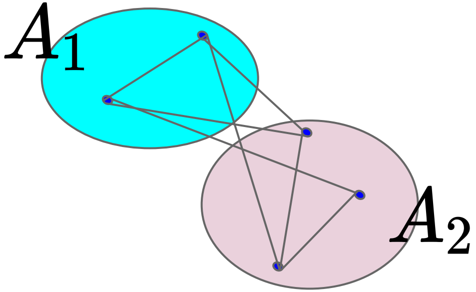

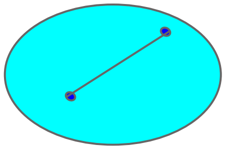

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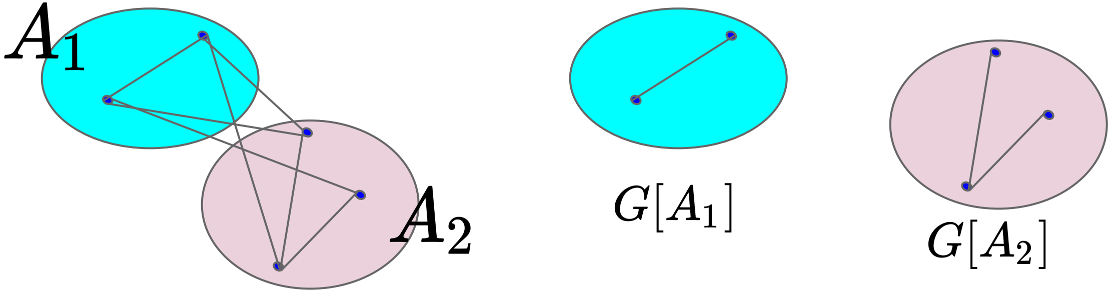

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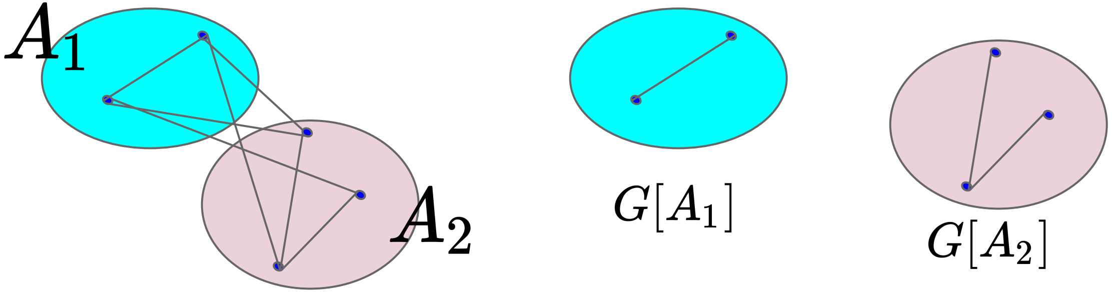

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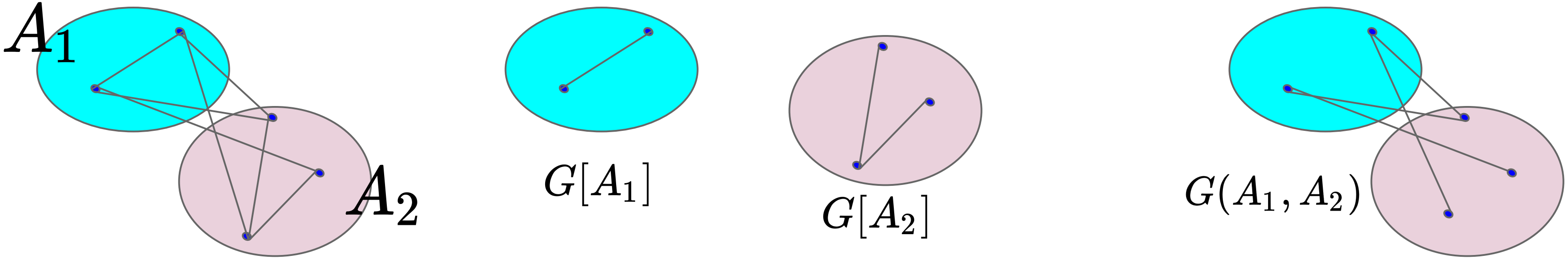

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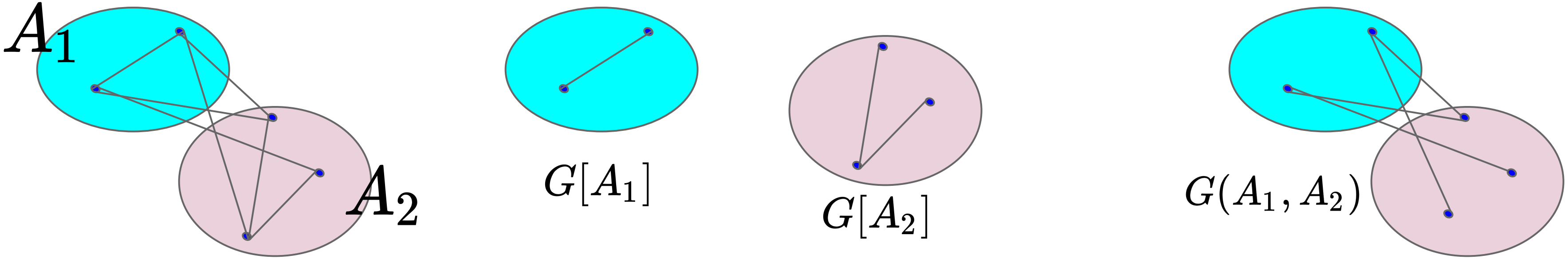

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A lower bound on d-dimensional algebraic connectivity

Let G=(V,E) be a graph, and $V = A_1 \cup \cdots \cup A_d$ a partition of its vertex set.

If $G[A_i]$ is connected for all $1 \le i \le d$ and $G(A_i, A_j)$ is connected for all $1 \le i < j \le d$, we call this a **strong d-rigid partition** of G.

Theorem (L-Nevo-Peled-Raz '22+):

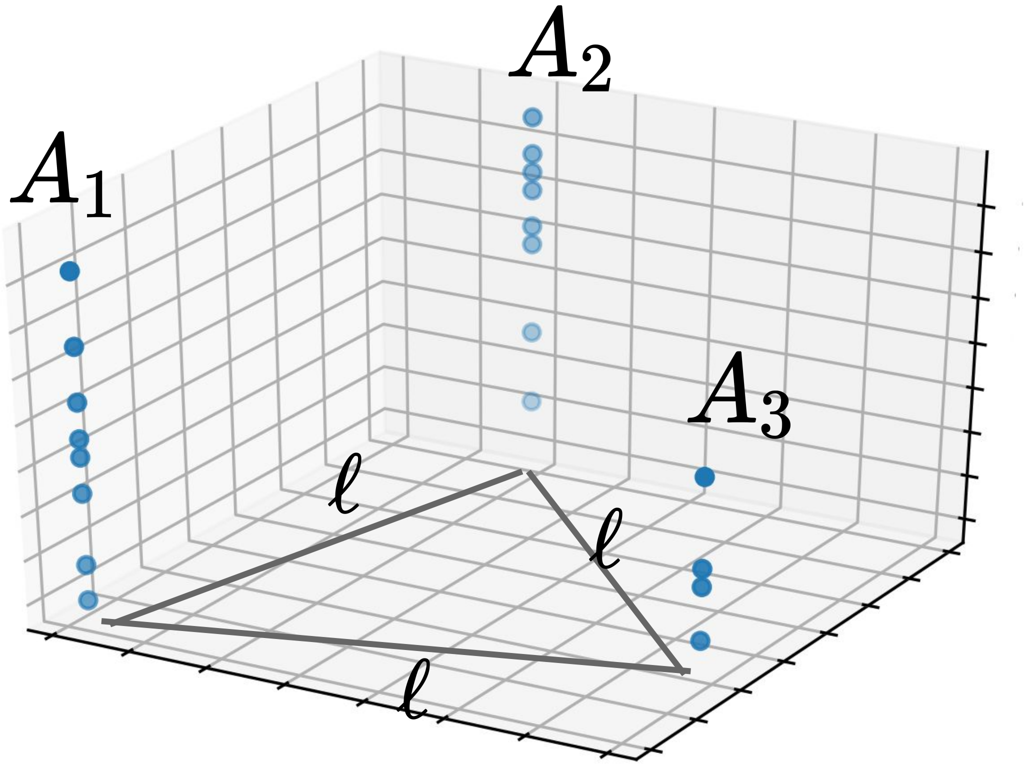
$$a_d(G) \ge \min\left(\{a(G[A_i])\}_{i=1}^{d} \cup \left\{\tfrac{1}{2}a(G(A_i, A_j))\right\}_{i<j}\right).$$

In particular, if G admits a strong d-rigid partition, it is d-rigid.

A lower bound on d-dimensional algebraic connectivity

$$d = 3$$

$$p =$$



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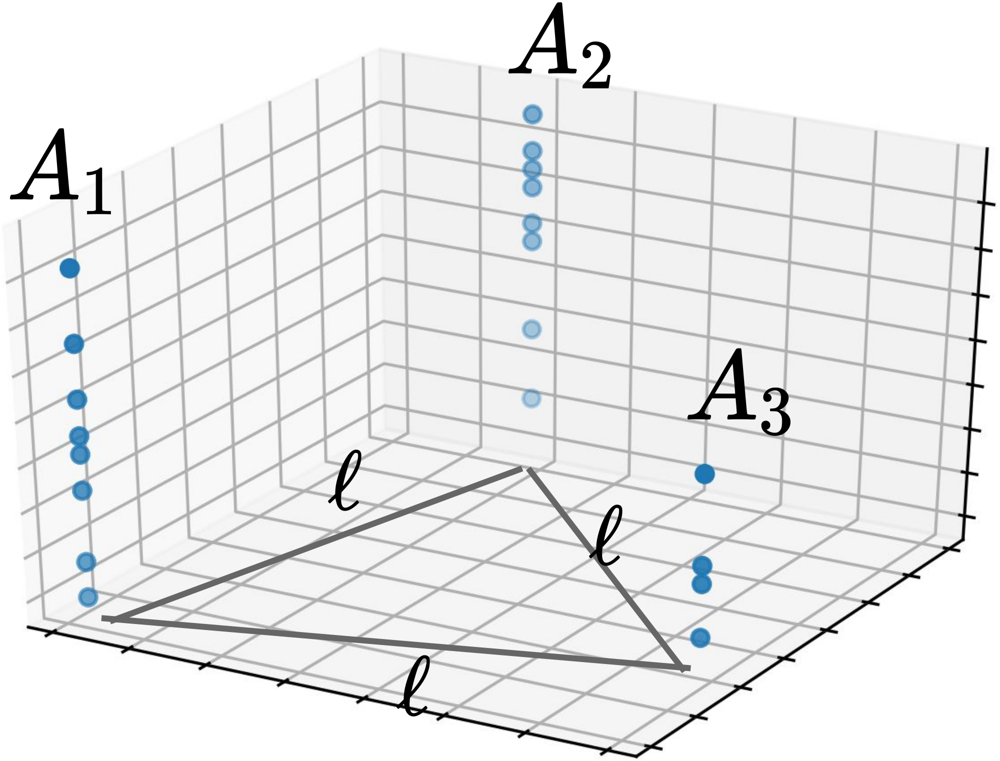

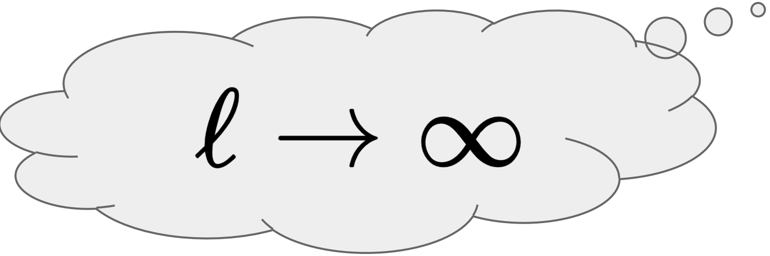

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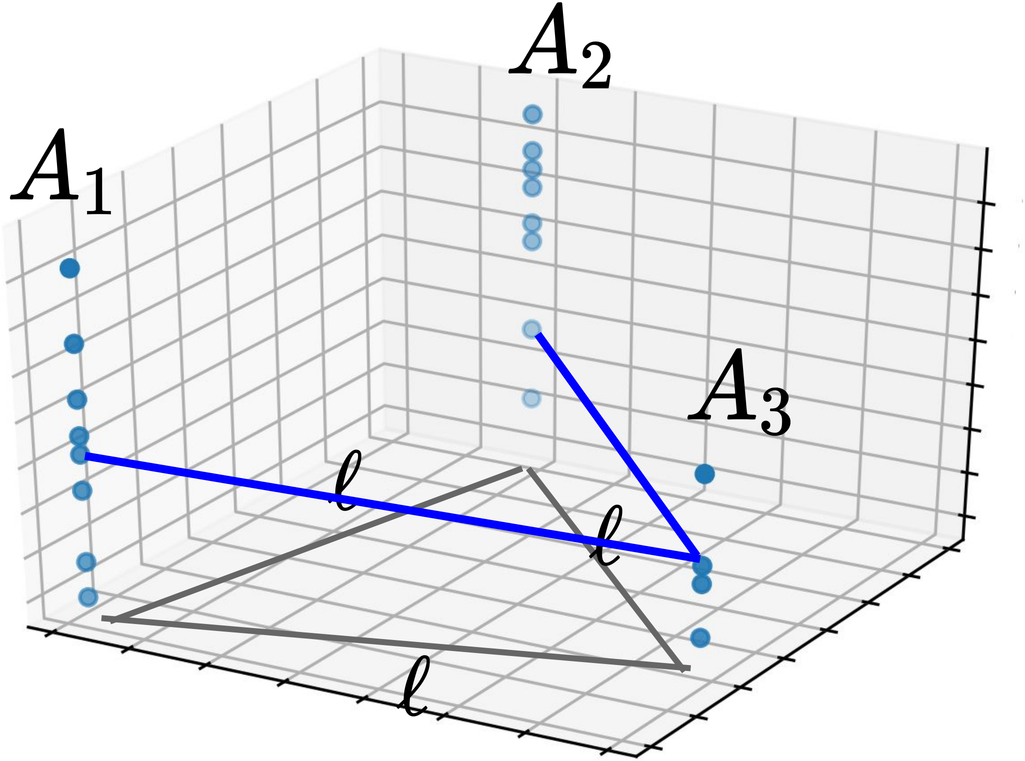

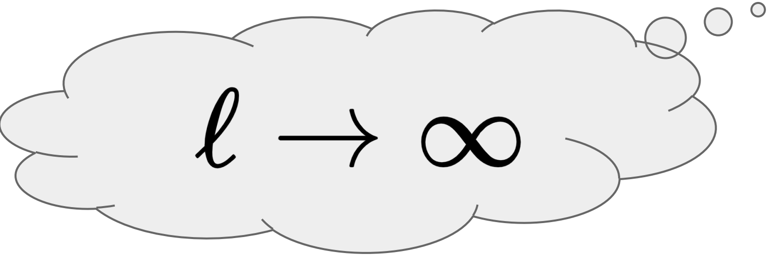

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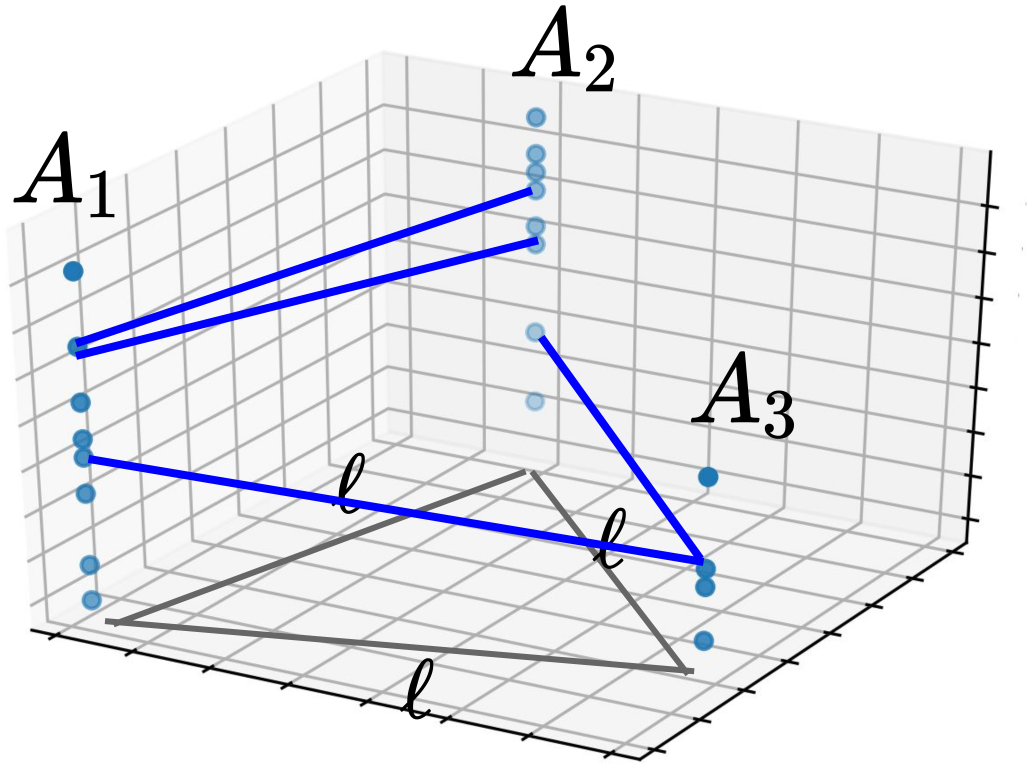

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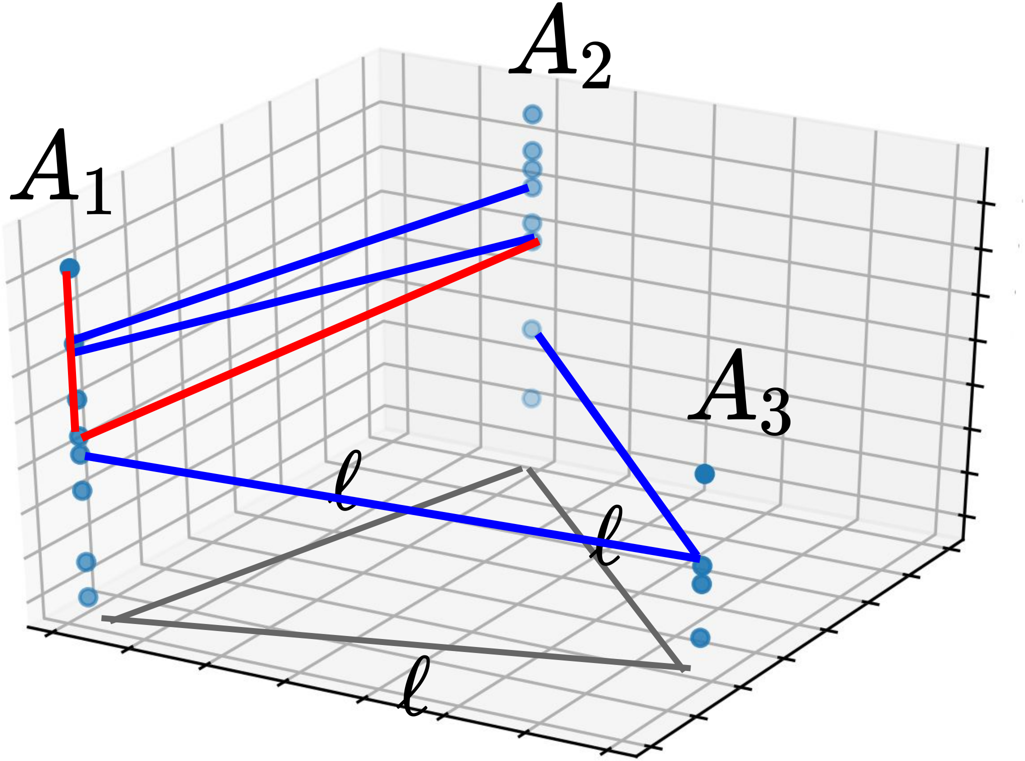

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Partition vertex set into d sets of size $\left\lfloor \frac{n}{d} \right\rfloor$ or $\left\lceil \frac{n}{d} \right\rceil$ each.

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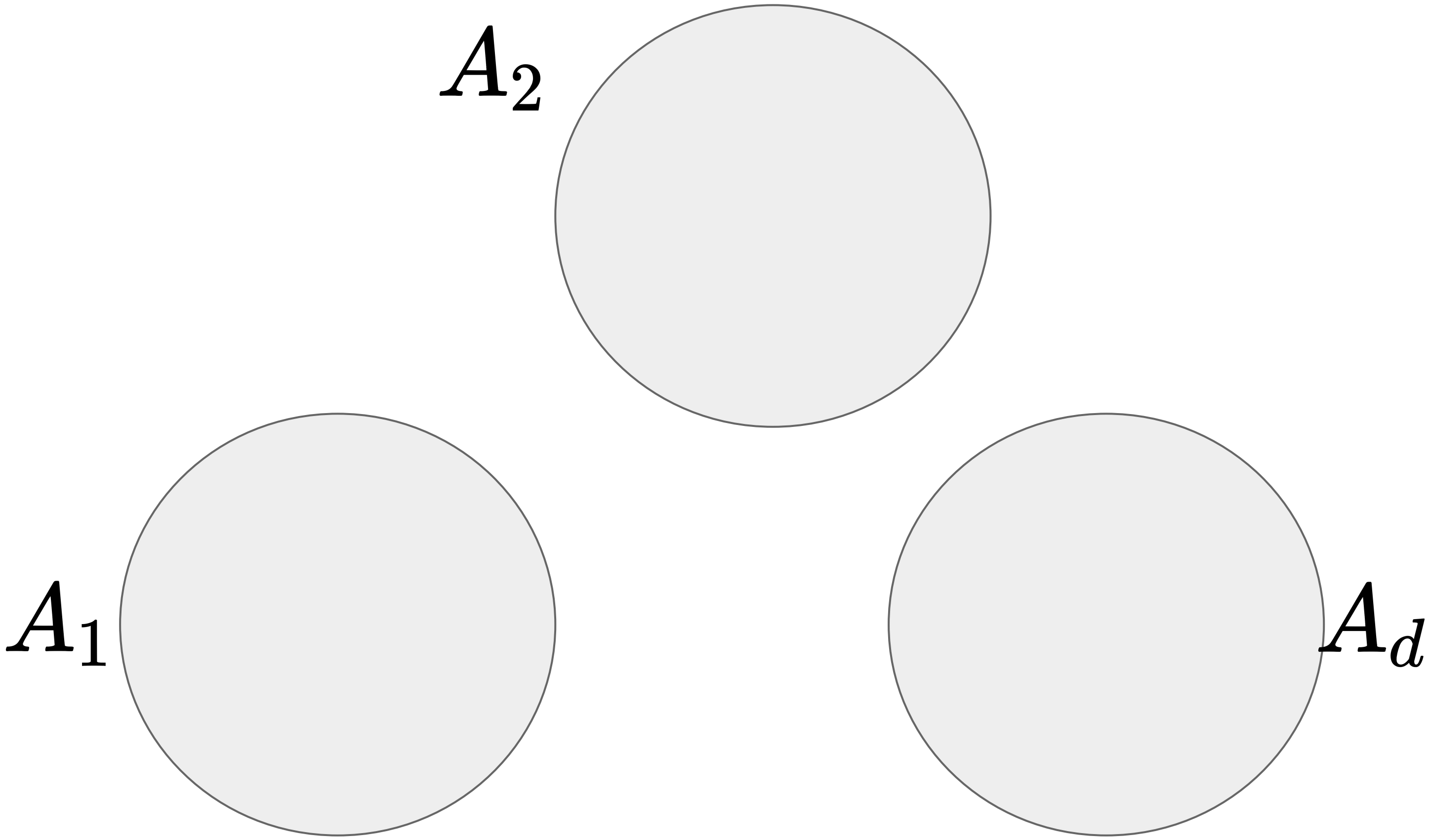

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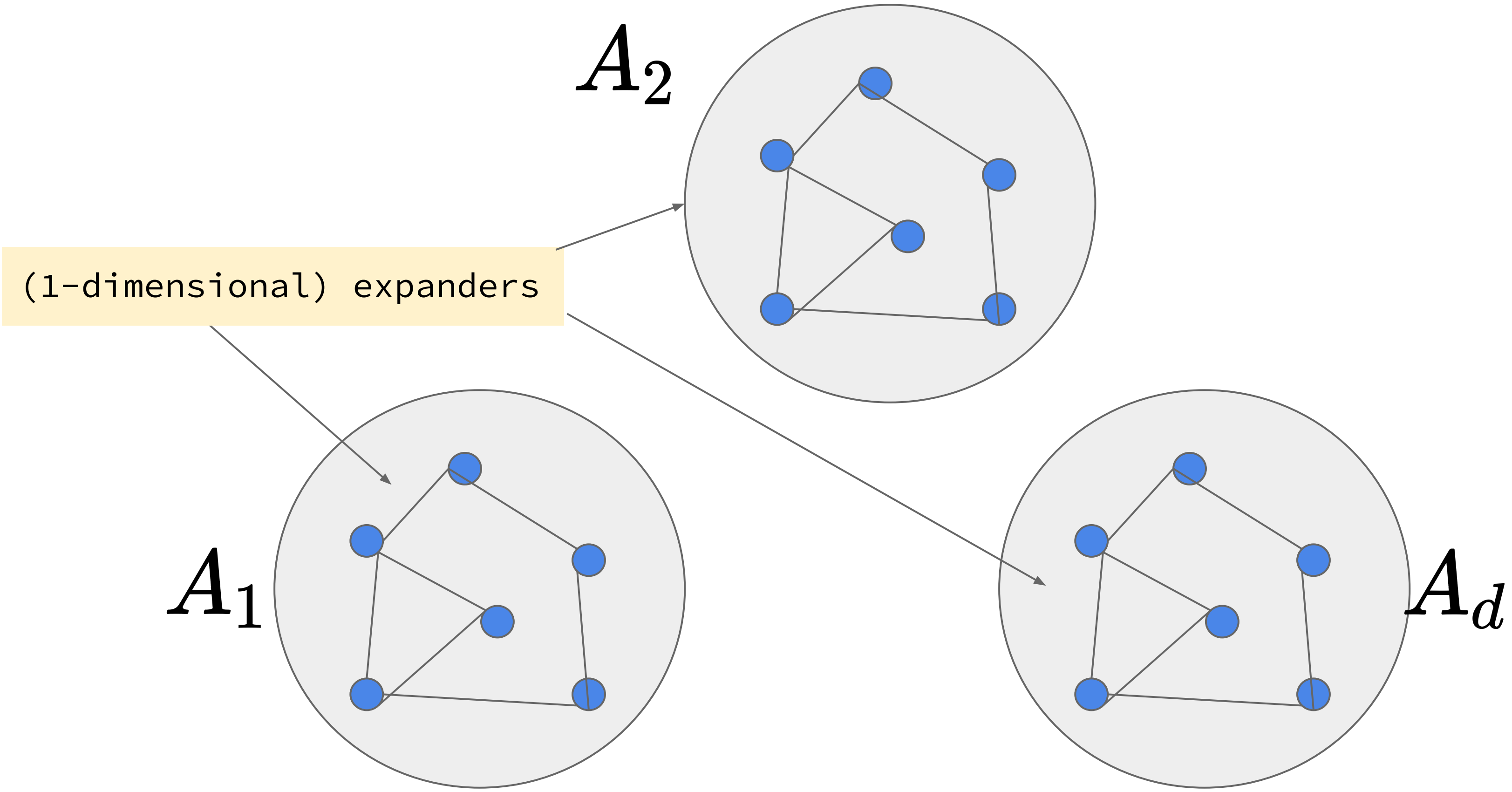

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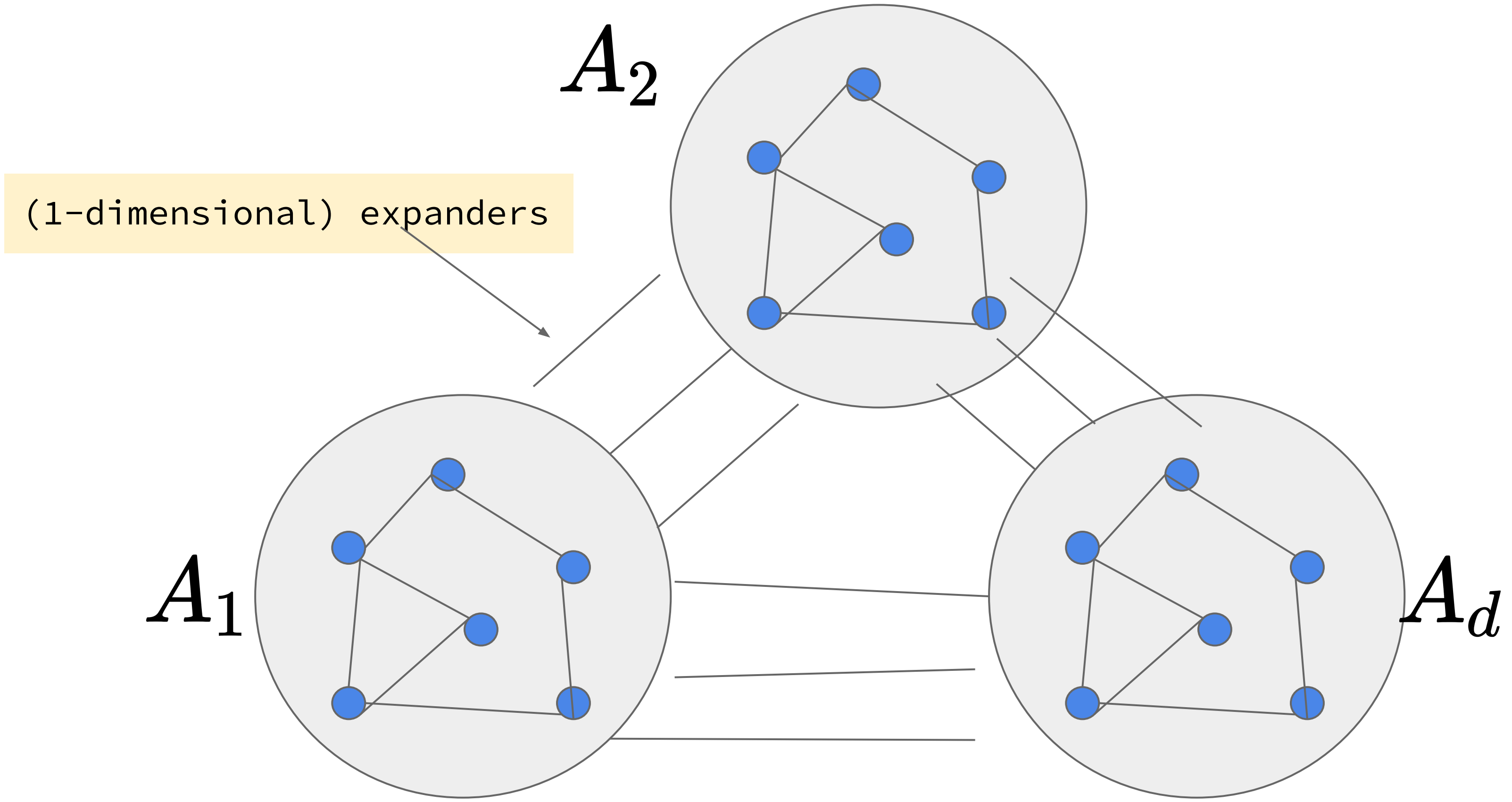
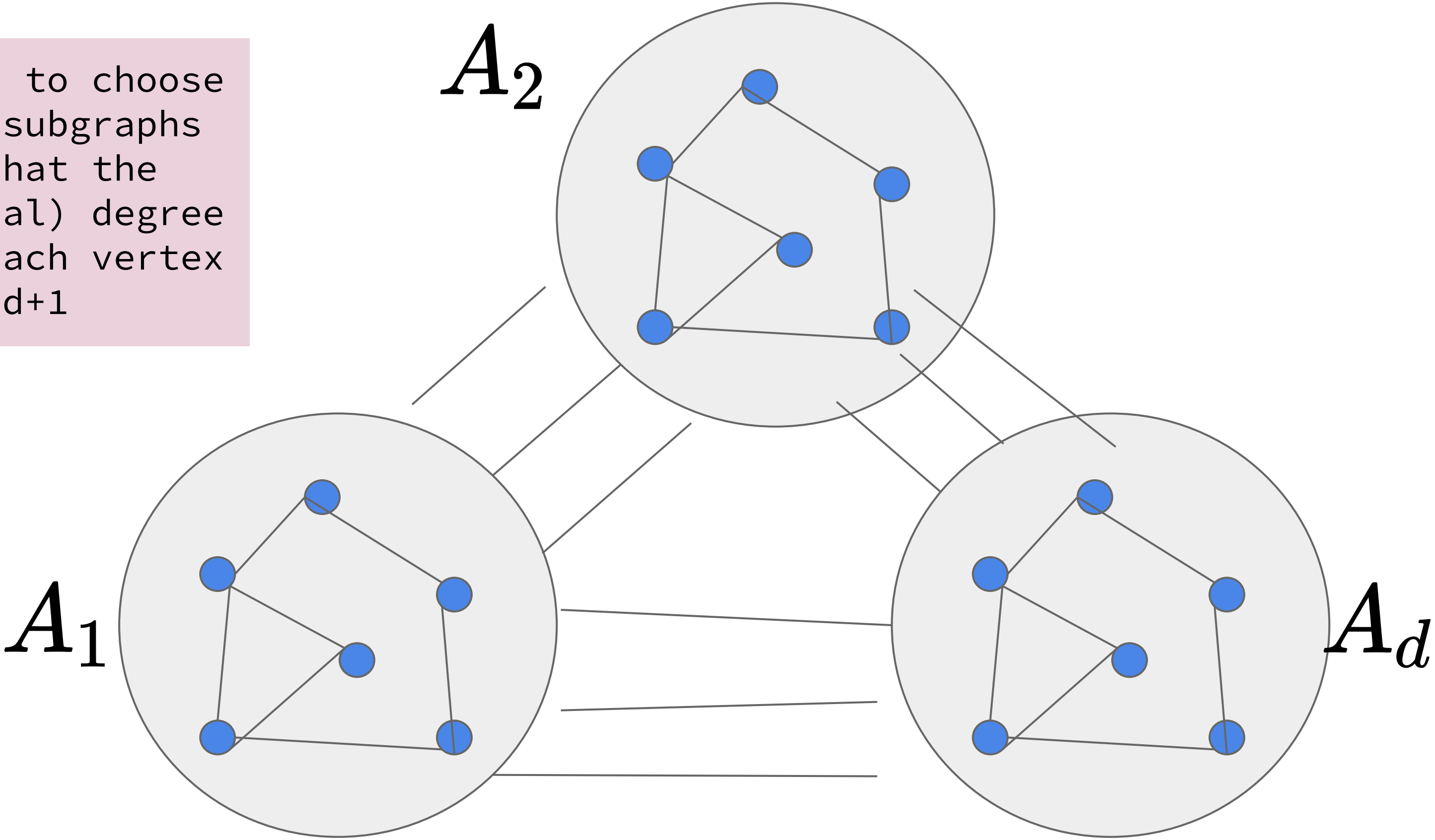


Construction of Rigidity Expanders

Need to choose the subgraphs so that the (total) degree of each vertex is 2d+1



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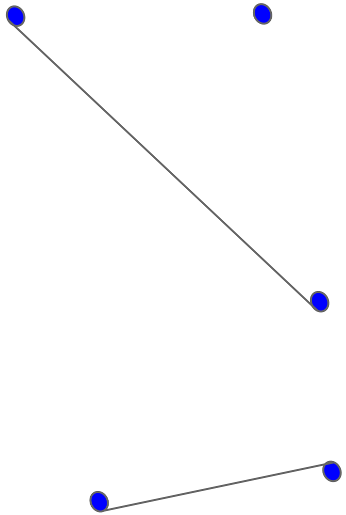

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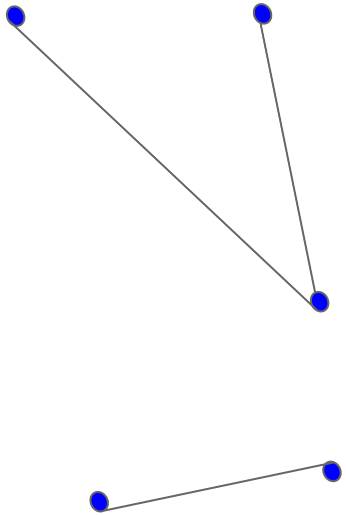

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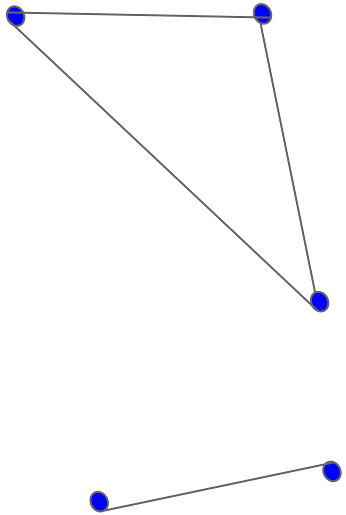

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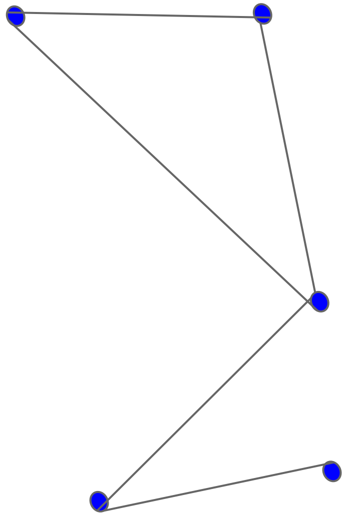

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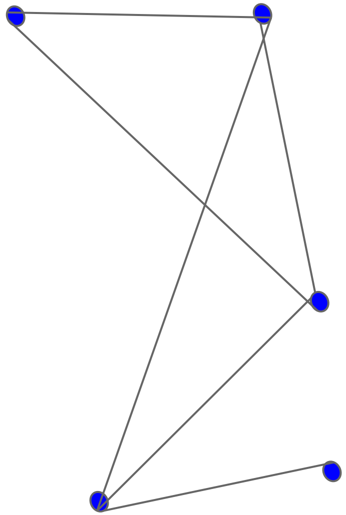

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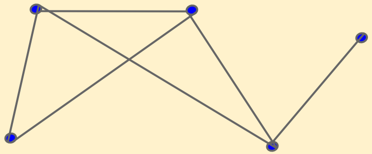

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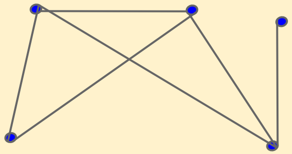

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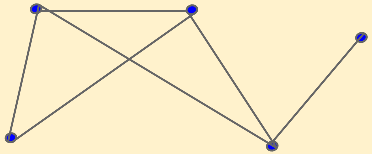

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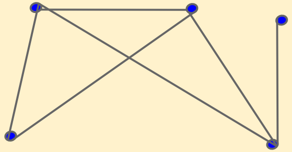

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More precisely:

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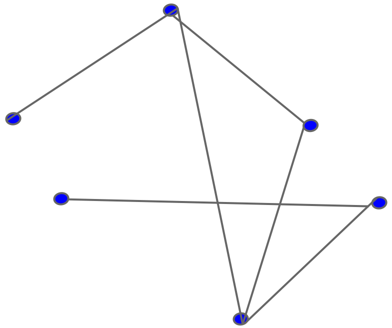

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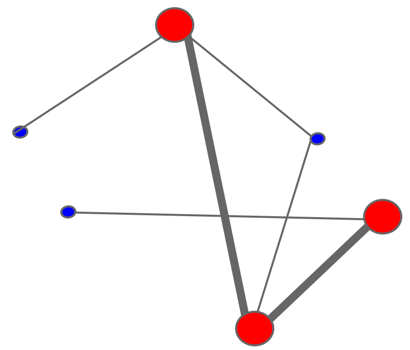

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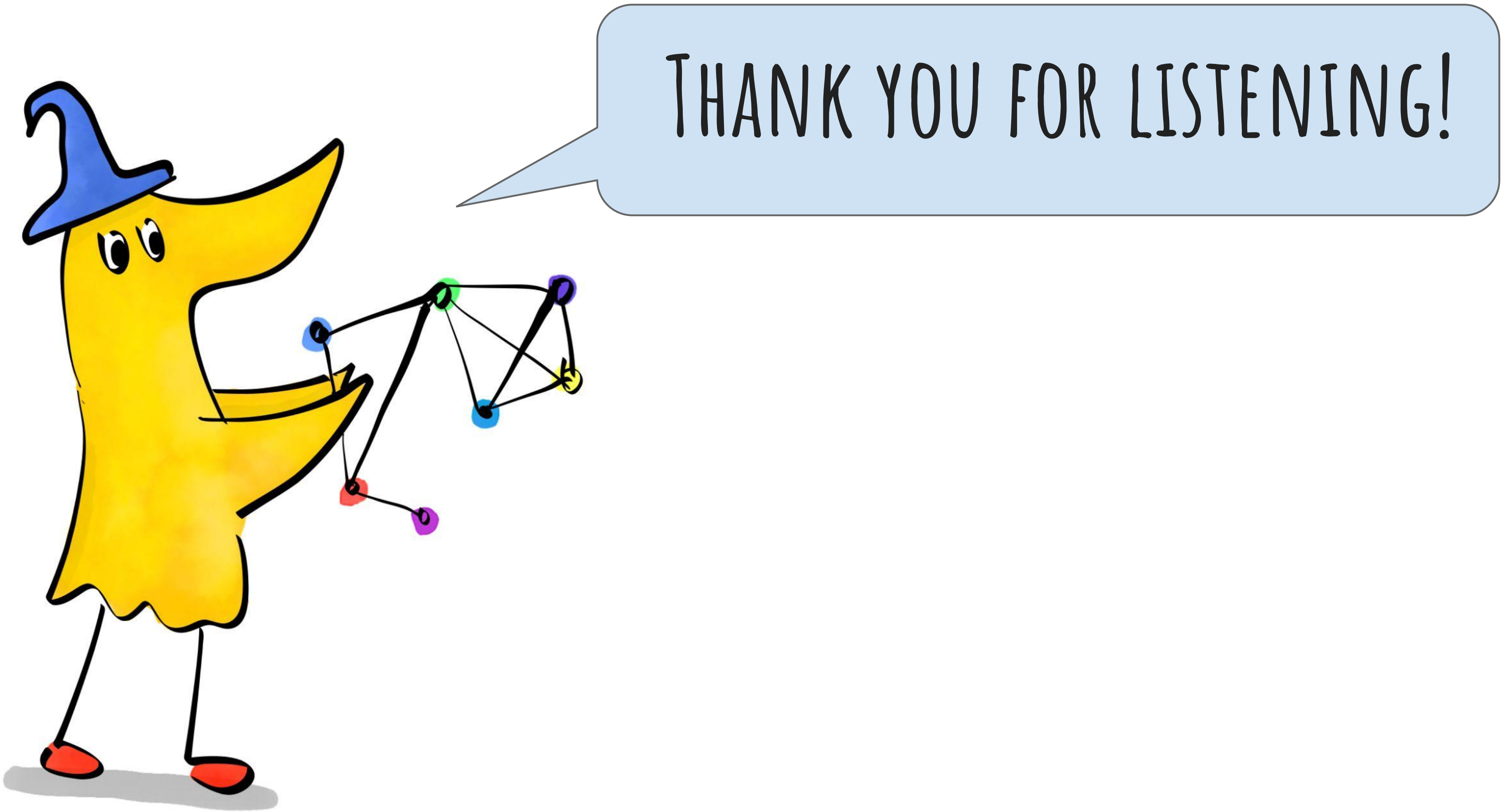
Thank you for listening!