21-122 - Integration and Approximation
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21-122 - Week 15, Recitation 1

Agenda

- Review: Polar Coordinates
- Correction 1: The meaning of $(-r, \theta)$
- Symmetry:
- Section 10.3: 11, 14, 19, 25, 37

Correction 1 - See page 654.
Example - Plot the point whose polar coordinates are $(-3, \frac{3\pi}{4})$.
Solution - The point lies in the fourth quadrant and has coordinates $(\sqrt{\frac{3}{2}}, -\sqrt{\frac{3}{2}})$. □

Correction 2 - The rose $r = \cos(k\theta)$ (k integer) has $2k$ petals if k is even and k petals if k is odd.

Symmetry
See page 659 for diagrams.

1. If a polar equation is unchanged when θ is replaced by $-\theta$, the curve is symmetric about the polar axis (x-axis).
2. If the equation is unchanged when r is replaced by $-r$ (equivalently, when θ is replaced by $\theta + \pi$), the curve is symmetric about the pole.
3. If the equation is unchanged by θ is replace by $\pi - \theta$, the curve is symmetric about the vertical line $\theta = \frac{\pi}{2}$.

Examples - The equation $r = \cos 2\theta$ represents a four-leaved rose (see page 658 or use Wolfram Alpha). This exhibits all three types of symmetry above.

Section 10.3
11. Sketch the region in the plane.

$$2 < r < 3, \qquad \frac{5\pi}{3} \leq \theta \leq \frac{7\pi}{3}$$

Solution - This is a segment of the ring of inner radius 2 and outer radius 3. See diagram in the back of the book. □
14. Find a formula for the distance between the points with polar coordinates (r_1, θ_1) and (r_2, θ_2).
Solutions - In Cartesian coordinates, the distance between two points (x_1, y_1) and (x_2, y_2) is

$$d = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$$

To find the distance in polar coordinates, let's convert our polar coordinates to Cartesian coordinates. In Cartesian coordinates, the points (r_1, θ_1) and (r_2, θ_2) can be written as

$$(r_1 \cos\theta_1, r_1 \sin\theta_1), \qquad (r_2 \cos\theta_2, r_2 \sin\theta_2)$$

Now the distance between these two points is

$$\begin{aligned} d &= \sqrt{(r_1\cos\theta_1 - r_2\cos\theta_2)^2 + (r_1\sin\theta_1 - r_2\sin\theta_2)^2} \\ &= \sqrt{r_1^2\cos^2\theta_1 + r_2^2\cos^2\theta_2 - 2r_1r_2\cos\theta_1\cos\theta_2 + r_1^2\sin^2\theta_1 + r_2^2\sin^2\theta_2 - 2r_1r_2\sin\theta_1\sin\theta_2} \\ &= \sqrt{r_1^2 + r_2^2 - 2r_1r_2(\cos\theta_1\cos\theta_2 + \sin\theta_1\sin\theta_2)} \\ &= \sqrt{r_1^2 + r_2^2 - 2r_1r_2\cos(\theta_1 - \theta_2)} \end{aligned}$$

□

19. Identify the curve $r^2\cos 2\theta = 1$ by finding a Cartesian equation for the curve.
Solution - Let's try and rewrite this in Cartesian coordinates. Write

$$1 = r^2\cos 2\theta = r^2(\cos^2\theta - \sin^2\theta) = (r\cos\theta)^2 - (r\sin\theta)^2 = x^2 - y^2$$

The Cartesian equation of the curve is $x^2 - y^2 = 1$. This corresponds to a hyperbola with center $(0, 0)$, foci on the x-axis, and asymptotes $y = \pm x$. □

25. Find a polar equation for the curve represented by the Cartesian equation $x^2 + y^2 = 2cx$.
Solution - Rewriting each side, we have $r^2 = 2cr\cos\theta$, or more simply, $r = 2c\cos\theta$. □

37. Sketch the curve $r = 2\cos 4\theta$ by first sketching the graph of r as a function of θ in Cartesian coordinates.
Solution - See diagram in the back of the book. □