Department of Mathematical Sciences
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21-366 Random Graphs
Test 3

You can use my book and you can quote theorems from the book.

Problem	Points	Score
1	25	
2	25	
3	25	
4	25	
Total	100	

Q1: (33pts)
Let $0 < p < 1$ be constant. Let $G = G_{n,p}$. Show that w.h.p. $G - H$ is connected for every subgraph H of G for which $\Delta(H) \leq np/3$. (Δ denotes maximum degree.)

Q2: (33pts)
Suppose that the edges e of K_n are given independent exponential mean one random lengths $\ell(e)$. Let $k \geq 2$ be a positive integer. Show that w.h.p. we can find edge disjoint paths $P_1, P_2, \ldots, P_k$ from 1 to 2 such that

$$\ell(P_1) + \ell(P_2) + \cdots + \ell(P_k) \approx \frac{k^2 \log n}{n}.$$

(Hint: randomly partition the edges of K_n into k sets.)

Q3: (34pts)
Let $p = \frac{c}{n}$ where $c < 1$ constant. Let X_k denote the number of directed cycles in $D_{n,p}$ with k edges.

1. What is $\mathbf{E}(X_k)$?
2. What is $\mathbf{E}(X_k(X_k - 1) \cdots (X_k - t + 1))$ for fixed t?
3. What is $\lim_{n\to\infty} \Pr(X_k = 0)$?

Q4: (34pts)

1. Show that in the Preferential Attachment Model,

$$\Pr(v_{t+1}\text{has a neighbor in } \{v_1, v_2, \ldots, v_{\lceil t/2 \rceil}\}) \geq \frac{1}{2}.$$

2. Hence, show that w.h.p. the Preferential Attachment Model with n vertices has diameter $O(\log n)$.