Homework 6: due November 11

1. Given $\mathcal{X}$ and an integer k we define the k-nearest neighbor graph $G_{k-NN,\mathcal{X}}$ as follows: We add an edge between x and y of $\mathcal{X}$ iff y is one of x's k nearest neighbors, in Euclidean distance or vice-versa. Show that if $k \geq C \log n$ for a sufficiently large C then $G_{k-NN,\mathcal{X}}$ is connected w.h.p.

2. A *tournament* T is an orientation of the complete graph K_n. In a random tournament, edge $\{u, v\}$ is oriented from u to v with probability $1/2$ and from v to u with probability $1/2$. Show that w.h.p. a random tournament is strongly connected.

3. Let T be a random tournament. Show that w.h.p. the size of the largest acyclic sub-tournament is asymptotic to $2 \log_2 n$. (A tournament is acyclic if it contains no directed cycles).