Optimal Inventory of a Campus Coffee Shop

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Abstract

In the context of starting a new coffee shop on campus, the study investigates an optimal inventory plan for an imaginary campus cafe that offers a representative small set of beverages that was selected according to the sales data of current campus cafes. The objective is to minimize the inventory costs while meeting the demands under the space and budget constraints. We aim to model the inventory cost for a campus coffee shop as a function involving quantity of orders. The basic idea comes from the inventory model we learned from class. We chose it because it is the elementary model which calculates the cost of order and cost of storage. We did some modifications based on our case study, changing the inventory cost to be the sum of order cost, inventory-related labor cost, transportation cost, and waste cost. We represent them separately in terms of quantity of orders. As we are investigating multiple items, we also combine multiple models (for each item) as our final model. The solution involves applying lagrange multiplier and integer programming for the final result. The result suggests reduced cost after optimizing the order quantities, and we also provide further analysis and discussions.

Introduction

As a group of sleep-deprived math majors who rely heavily on caffeine to function, we have always been despondent to see the perpetual long lines at the CMU coffee shops such as La Prima, De Fer, and Red Hawk. It soon occurred to us that this need isn't just ours, but a collective demand among CMU students. To tackle this directly, we believe opening up a new coffee shop on campus could be an effective solution. In addition to meeting the demands, a new coffee shop option for students will also encourage price competition among shops and address the current overpriced menus. In taking this initiative, we surveyed the campus cafes on their operations, and figured that an inventory strategy is an essential piece of this business. Thus, we decided to kick off our ambitious plan of offering more accessible and affordable drink options by investigating the optimal inventory management of a campus coffee shop.

Assumptions

In our analysis of the coffee shop's operations, we have placed foundational trust in the authenticity of the data provided by the campus coffee shop owner and employees. This includes spreadsheets, surveys, and interviews, which we've used to estimate data for an individual shop based on averages from all the coffee shops they operate. Our operational assumptions are modeled after a reference coffee shop, operating six days a week and four weeks a month. We observed stable customer demand in the data, leading us to simplify our approach by assuming constant demand.

Costs and constraints also play a crucial role in our analysis. We noted that CMU does not directly incur storage, rent, or utility costs, and due to limited storage space, we combined the capacities of both fridge and room storage. For the drink compositions, we standardized measurements:

- A cup of Americano is made up of one shot of espresso and uses one disposable cup
- A cup of Latte is made up of one shot of espresso, ten ounces of milk, and uses one disposable cup
- A cup of Mocha is made up of one shot of espresso, ten ounces of milk, one pump of chocolate syrup, and uses one disposable cup

These assumptions are key in providing a realistic and actionable framework for our analysis.

Method and Models

Part 1: basic model

We start with the basic model covered in class.

Total cost CT(Q) = total order cost + total storage cost

$$= \frac{K}{T} + \frac{IQ}{2}$$

In this model, K stands for the fixed total cost associated with making orders, Q stands for the quantity of order, T stands for the time interval between any 2 orders, I stands for the inventory carrying charges.

We then introduce the gradient v (the rate of using up stock), which is calculated as Q = vT. So we rewrite the model as:

Total cost CT(Q) = $\frac{Kv}{Q} + \frac{IQ}{2}$

The most important part of this model is how it handles the order cost. Notice that the expression $\frac{K}{T}$ (or $\frac{Kv}{Q}$) counts the amount of order cost within one period for one item.

Part 2: our model

In our case of study, we want to implement a new model to the coffee shop's inventory system, so that the total inventory cost could be covered.

After communicating with the shop owner and employees, we summarized several relevant costs. However, since the campus coffee shop does not pay rent (covered by university), we do not consider storage cost in our model.

- TotalCost = order cost + inventory-related labor cost + transportation cost + waste cost

$$C_T(Q) = C_{order} + C_{Inventory} + C_{Transport} + C_{Waste}$$

- Define parameters
    - K = Fixed order cost
    - Q = Quantity of order (per time)
    - v = gradient of Q/T (rate of using up stock)
    - a = Amount of stock handled within a hour
    - C_{Labor} = labor cost per hour
    - b = percentage waste
    - s = unit cost per import quantity

With these parameters, we formulate the costs:

- C_{order} = total order cost per time: $\frac{Kv}{Q}$
- $C_{Inventory}$ = Labor cost related to organizing inventory: $\frac{Q}{a} C_{Labor}$
- $C_{Transport}$ = transportation cost (fixed) = some constant P
- C_{Waste} = bQs

Then:

$$C_T(Q) = C_{order} + C_{Inventory} + C_{Transport} + C_{Waste}$$
$$= \frac{Kv}{Q} + \frac{Q}{a} * C_{Labor} + P + bQs$$

where $C_T(Q)$ represents the total inventory cost for one ingredient.

Here, the minimum total cost value corresponds to the minimum value of the function $C_T(Q)$. The ordered quantity that minimizes the cost in inventory management is defined as Q^*. We determine Q^* by setting the equation's first derivative to 0.

$$C_T'(Q) = -\frac{Kv}{Q^2} + \frac{C_{laber}}{a} + bs$$

$$0 = -\frac{Kv}{Q^2} + \frac{C_{laber}}{a} + bs$$

$$Q^* = \sqrt{\frac{Kv}{\frac{C_{laber}}{a}+bs}} = \sqrt{\frac{aKv}{C_{laber}+abs}}$$

(We call Q^* the unconstrained optimal quantity of order for one item)

Since we are considering three menu items (Americano, Latte, and Mocha), we investigate their four ingredients (coffee beans, milk, chocolate syrup, and disposable cup) in the inventory system.

Assume C_{Labor} and P remain constant for each ingredient, we sum the four items up to obtain our final model:

$$C_{All} = \sum_{i=1}^{4} \left(\frac{K_i D_i}{Q_i} + \frac{Q_i}{a} * C_{Labor} + b_i Q_i s_i + P\right)$$

Part 3: constrained model

While campus coffee shops do not have storage cost, they do have a storage space limit. After investigation and information from the staff, the space constraint is estimated to be approximately 50 ft^3.

We denote the space that each unit of ingredient takes as f_i (data in next section), and denote the total space as F = 50 ft^3 = 1.41584 m^3.

$$F = \sum_{i=1}^{4} f_i Q_i$$

Thus we have an objective and its constraint:

$$\text{Minimize } C_{All} = \sum_{i=1}^{4} \left(\frac{K_i D_i}{Q_i} + \frac{Q_i}{a} C_{Labor} + b_i Q_i s_i + P\right)$$

$$\text{Subject to } F = \sum_{i=1}^{4} f_i Q_i = 1.41584$$

In order to solve this problem, we apply the Lagrange Multiplier Technique and denote the new optimal quantity by $Q^{*}_{i,\lambda}$:

$$\nabla C = \lambda \nabla F$$

$$\frac{\partial C}{\partial Q_i} = \lambda \frac{\partial F}{\partial Q_i}$$

$$-\frac{K_i v_i}{Q_i^{2}} + \frac{C_{Labor}}{a_i} + b_i s_i = \lambda f_i$$

$$Q^{*}_{i,\lambda} = \sqrt{\frac{a_i K_i v_i}{C_{Labor} + a_i b_i s_i - a_i \lambda f_i}}$$

(We call $Q^{*}_{i,\lambda}$ the constrained optimal quantity of order for item i)

Then, to obtain the value of λ, we insert $Q^{*}_{i,\lambda}$ into the constraint function:

$$\sum_{i=1}^{4} f_i \sqrt{\frac{a_i K_i v_i}{C_{Labor} + a_i b_i s_i - a_i \lambda f_i}} = 1.41584$$

which solves λ.

Data

The data collection phase required a multifaceted approach. In addition to visiting the campus coffee shops regularly to collect information, we conducted interviews with employees at existing campus coffee shops, gaining insights into their inventory practices and salaries. Surveys were distributed among students to estimate monthly beverage preferences and demands, and meetings with shop managers shed light on more operational details such as the cost of raw materials and transportations.

Instead of focusing on the three beverages themselves, we chose to focus on the main ingredients of these products, i.e. the coffee beans, milk, chocolate syrup, and disposable cups.

The unit we used for the ingredients are as follows:

- coffee beans - one gram
- milk - one ounce
- chocolate syrup - one pump or half ounce
- disposable cups - one cup

Based on the recipe of the three types of coffee we mentioned in the assumption part before, we gathered the initial quantity of order (Q) for each ingredient per order. Given the information provided by the campus coffee shop, we also gained the monthly total order cost (K) and the days between orders (T) for ingredients. Based on the information available, we computed the gradient per period (v), which is measured in terms of quantity of order per day. Furthermore, by interviewing current employees of the campus coffee shop, we calculated and estimated the percentage of waste (b) for these ingredients as well. We also collected and computed the data of handle rate per hour (a), which is the amount of each raw material the employees are able to organize in one hour, and the unit import price (s), which indicates the import price for one unit of the ingredient. Finally, we collect the space taken for each unit item (f) by estimating through online data. The specific data is shown in table 1 below.

In addition to these collected data, we also obtained some important information from the campus coffee shop that the labor cost related to the transportation and distribution of raw materials is approximately $15/hr, and the total monthly transportation cost is $100. Moreover, we knew that the total storage cost was 0.

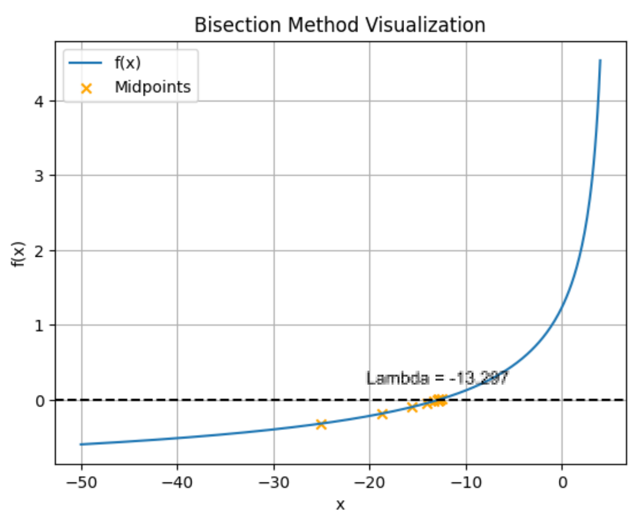
Table 1. Data

Items	Quantity of Order (Q)	Total order cost (K)	Period of order (T)	Percentage waste (b)	Gradient per period (v)	Handle rate / hr (a)	Unit Import price (s)	Estimated space taken (f)
Coffee Bean (g)	1496.59	$40.00	6	3%	249.43	5327.5	$0.022	0.000000833333333
Milk (oz)	353.325	$60.00	2	5%	176.66	740	$0.0488	0.00003125
Chocolate Syrup (pump)	23.46	$8.00	6	2%	3.91	1368	$0.1	0.000025
Disposable Cup (cup)	897.9025	$32.00	6	3%	149.65	4795	$0.09	0.001215

Result Analysis and Conclusion

1. Quantity of orders

To find the unconstrained optimal order quantity, we solve the equation for each item. Then, to find the constrained optimal order quantity, we solve the equation for each item, and determine λ by solving the nonlinear equation using numerical methods.

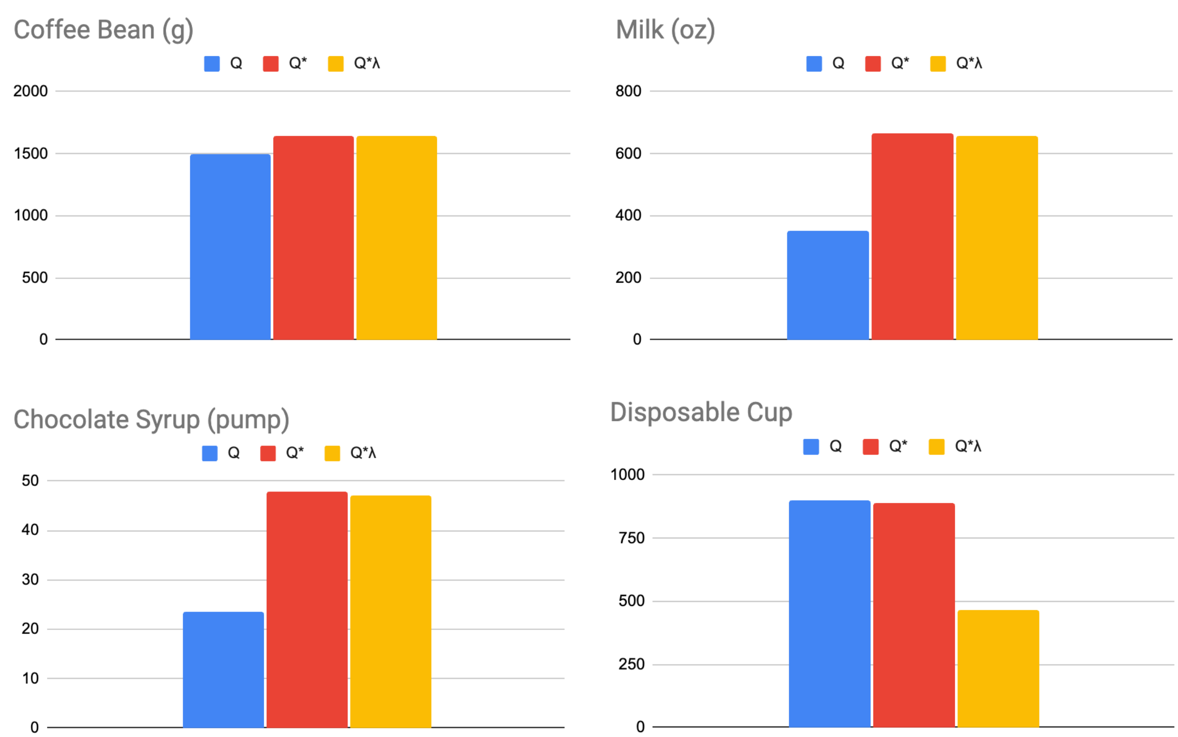


The table below summarizes the optimal order quantity, with and without storage space constraint, per period our model suggests for each of the four items along with the corresponding costs. Through our model, we obtained the unconstrained optimal cost that is the lowest among the three, followed by the constrained optimal cost. We observe that the change in order quantity from our model suggested an increase in all items but disposable cups. The drastic decrease in disposable cup orders from out unconstrained to constrained optimization could indicate that the storage space each unit of disposable cup takes up is significantly larger than this ratio for other items.

Table 2. Order Quantity Comparisons

Item	Reference quantity Q	Unconstrained optimal quantity Q^*	Constrained optimal quantity Q^*_λ
Coffee Beans (g)	1496.5900	1650.3287	1647.8383
Milk (oz)	353.3250	663.7207	658.0628
Chocolate Syrup (0.5 oz)	23.4600	47.7901	47.2205
Disposable Cups	897.9025	890.6565	464.5252

Figure 1. Visualization of Order Costs Comparisons



2. Total inventory cost

Upon applying our optimized ordering quantities to the total cost calculation,we found that the overall cost has indeed decreased. When considering the space constraint, the total cost was marginally higher compared to the scenario without space constraints. This slight increase in cost is likely attributable to the necessity of more frequent ordering to accommodate the limited storage space.

Table 3. Order Costs Comparisons (in US dollars)

Item	Reference cost/period	Unconstrained optimal cost/period	Constrained optimal cost/period
Coffee Beans (g)	48.5964	47.9591	48.3650
Milk (oz)	462.0189	218.2812	223.5909
Chocolate Syrup (0.5 oz)	6.6186	2.5208	2.6183
Disposable Cups	43.0152	53.8651	104.9064
Monthly Sum	560.2490	322.6262	379.4807

Improvements and Suggestions for Future Work

There are several limitations in our study that could be improved with a more sophisticated data collection process. Our data is heavily centered around a few campus coffee shops at CMU, which might not be representative of all campuses of all universities. We were also able to acquire only two months of sales data, limiting the accuracy of our demand estimates. Furthermore, a portion of the inventory-related cost data, including delivery and fixed order costs, was collected through interviews with the current employees and the shop owner. These are based on their estimations, the validity of which we were not able to verify.

Moving beyond the current limitations of our study caused by the data collection process, we propose some possible modifications to our model for further investigation. Firstly, the only constraint considered in this study is the storage space constraints of the four items we consider. However, during our conversations with the data provider, we identified other relevant constraints, including van space for delivery and monthly budget limitations. Secondly, we based our model on the coffee shop's claim of no backordering situations in their operations. This might not be the usual case for a coffee shop in general, so incorporating this factor in future work could build a more robust and generalizable model. Furthermore, we could include delivery time considerations in our model, as we currently assume instant delivery of orders. Thirdly, we assumed a constant demand over time in our model, which could deviate from reality. Improvements in modeling demand might include applying the PERT estimate, which involves analyzing the best, worst, and average case scenarios, or simulating randomized demand. Lastly, as our ultimate goal is to construct a business plan for a new campus coffee shop, we might want to optimize overall profit instead of focusing purely on minimizing inventory-related costs. This would expand our model to include considerations such as trade-offs between storage and seating spaces, hiring plans for employees, and how those decisions might impact the demand and supply of our coffee shop. These improvements aim to address the limitations and broaden the scope of our study, contributing to a more detailed and applicable business plan for campus coffee shops.

Reference

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