Department of Mathematical Sciences
CARNEGIE MELLON UNIVERSITY

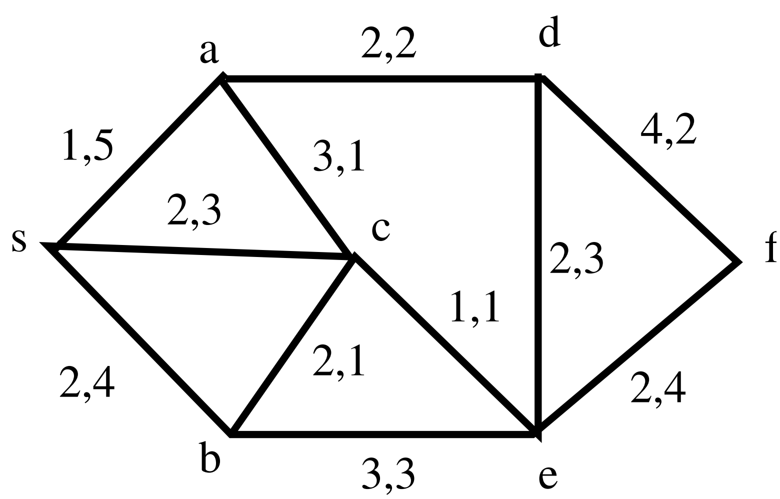
OPERATIONS RESEARCH II 21-393

Homework 2: Due Monday October 9.

Q1 Solve the following 2-person zero-sum games:

$$\begin{bmatrix} 6 & 2 & 4 \\ 5 & 2 & 5 \\ 4 & 1 & -3 \end{bmatrix} \qquad \begin{bmatrix} 2 & 1 & 1 & 0 & -1 \\ 4 & 3 & 2 & 1 & -1 \\ 1 & 1 & 0 & -1 & 1 \\ 2 & 1 & 1 & -2 & -2 \\ 4 & 1 & 0 & -2 & -3 \end{bmatrix}$$

Q2 Find a shortest path from s to all other nodes in the digraph below. Each edge (x, y) is labelled by a pair (a, b) and the length of the corresponding arc is $a + bt$ where t is the time the path reaches x. All arcs are directed lexicograhically e.g. (c, e) is directed from c to e.



Q3 There are two machines available for the processing of $n = 2m$ jobs. The processing time of job j is $p_j > 0$ for $j = 1, 2, \ldots, n$. The objective is to assign jobs to machines in order to minimise $\sum_{j=1}^{n} C_j$ where C_j is the completion time of job j.

(a) Suppose that in an optimum schedule machine 1 processes jobs $i_1, i_2, \ldots, i_s$ and machine 2 processes jobs $j_1, j_2, \ldots, j_t$ in this order. Show that the contribution of machine 1 to the objective function is

$$sp_{i_1} + (s-1)p_{i_2} + \cdots + 2p_{i_{s-1}} + p_{i_s}.$$

(b) Show that $p_{i_1} \leq p_{i_2} \leq \cdots \leq p_{i_s}$.

(c) Show that $s = t = m$ in the optimal solution.
(Hint: if $s \geq m+1$, see the effect of moving job i_1 to the front of machine 2's list.)

(d) Show that $p_{i_m} \geq p_{j_{m-1}}$.

Deduce the structure of an optimal solution.