Department of Mathematical Sciences
CARNEGIE MELLON UNIVERSITY

OPERATIONS RESEARCH II 21-393

Homework 2: Due Monday September 25.

Q1 Solve the infinite horizon problem for the given matrix of costs. Assume that $\alpha = 1/2$.

$$\begin{bmatrix} 5 & 4 & 1 & 8 \\ 2 & 1 & 5 & 6 \\ 3 & 1 & 5 & 4 \\ 4 & 3 & 6 & 1 \end{bmatrix}$$

Begin with the policy

$$\pi(1) = 4, \pi(2) = 4, \pi(3) = 3, \pi(4) = 4.$$

Q2 Solve the following problem by a cutting plane algorithm:

$$\begin{array}{lllll} \text{minimise} & 4x_1 + 5x_2 + 3x_3 & & & \\ \text{subject to} & & & & \\ & 2x_1 + x_2 - x_3 & \geq & 2 \\ & x_1 + 4x_2 + x_3 & \geq & 13 \end{array}$$

$$x_1, x_2, x_3 \geq 0 \text{ and integer.}$$

Q3 An assembly line consists of a sequence of locations called work stations. The manufacture of a certain object requires m separate jobs to be undertaken with job i requiring t_i minutes. The jobs are to be allocated to work stations so that each station completes a set of jobs and then passes the object onto the next station on the line and waits to receive the next object from the previous station on the line. The combined time of all jobs assigned to any station must not exceed T the cycle time. Also there are a number of precedence relations between jobs indicated by the digraph $D = (V, A)$ where $(i, j) \in A$ if job i must precede job j. The problem is to open as few work stations as possible consistent with the cycle time. Formulate this as an integer programming problem.
(Hint: let $x_{i,j} = 1$ iff job j is done on station i and $y_i = 1$ if there is at least one job assigned to station i.)