Department of Mathematical Sciences
CARNEGIE MELLON UNIVERSITY

OPERATIONS RESEARCH II 21-393

Homework 1: Due Monday September 11.

Describe a Dynamic programming solution to the following problems:

Q1 A company manufactures two products A and B at a certain facility. The demands for the products are $a_i, b_i, i = 1, 2, \ldots, n$ over the next n periods. The cost of making x of either product is $c(x)$ and there is room to store H in total of the two products. Cleaning problems require that only one product can be manufactured in any one period. Assume that at the beginning of period one there is $H/2$ of each product in storage. The problem is to minimise total cost, given that all demands must be met.

Q2 You have to drive across country along a road of length L. There are gas stations at points $P_1, P_2, \ldots, P_r$ along the route. Your car can hold g gallons of gasoline. At gas station i, the price of gas is p_i per gallon. If you drive at s miles per hour then you use up $f(s)$ gallons of gas per mile. You can assume that you have to drive at constant speed between stops. You start with a full tank of gas and you have an amount A to spend on the trip. Can you finish the trip in time at most T?
Hint: let $f(i, a, \gamma)$ denote the minimum time to get from P_i to P_r given you are at P_i, you have a miles to go and γ gallons in your car. Find a recurrence for f.

Q3 The people of a certain area live at the side of a long straight road of length L. The population is clustered into several villages at points $a_1, a_2, \ldots, a_n$ along the road. There is a proposal to build ℓ fire stations on the road. The problem is to build them so that the maximum distance of a village to its nearest fire station is minimised. Formulate the problem of finding the optimum placement of fire stations as a dynamic program. (Assume that fire stations are to be placed at integer points only on the line.)
Hint: for an interval $I = \{i, i+1, \ldots, j\}$ let $d(I, k)$ denote the maximum distance to a fire station placed at k from villages placed in I. Let

$D(I) = \min_{k \in I} d(I, k)$. Now break up stick of length L into ℓ intervals $I_1, I_2, \ldots, I_\ell$ and minimise $\max\{D(I_j) :_j = 1, 2, \ldots, \ell\}$.