Department of Mathematics
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Discrete Mathematics 21-228

Answers to homework 9.

Q1: (1) and (2) imply (3): Remove an edge from the unique cycle. Now the graph is connected and acyclic. It is a tree and so has $n-1$ edges ($n = |V|$). Thus G has n edges.

(1) and (3) imply (2): The graph is not a tree (too many edges) and so must contain a cycle. Remove an edge e of this cycle. Now we have a connected graph with $n-1$ edges. This is now a tree and adding back e produces a unique cycle.

(2) and (3) imply (1): Remove an edge e from the unique cycle. What remains is an acyclic graph with $n-1$ edges. This is a tree, which is connected. Thus G itself is connected.

Q2: Add e to T_2 and remove an edge $f \neq e$ from the cycle created. Put $T_3 = T_2 + e - f$.

Q3: Let $G - v$ have components $K_1, K_2, \ldots, K_r$. The neighbours of v in K_i have odd degree in K_i. Thus v has at least 2 distinct neighbours in each K_i and $r \leq d(v)/2$.