Discrete Mathematics 21-228: Spring 2002
Homework.
HW7 due Friday 3/22/02

Q1: Let b_n denote the number of ways of partitioning $[n]$ into sets of size 1 or 2.

(a) Let $b_0 = 1, b_1 = 1$. Prove the recurrence

$$b_n = b_{n-1} + (n-1)b_{n-2} \qquad n \geq 2.$$

(b) Let $b(x) = \sum_{n=0}^{\infty} b_n x^n/n!$. Show that b satisfies the differential equation

$$b'(x) = (1+x)b(x).$$

Hint: multiply the recurrence by $x^{n-1}/(n-1)!$ and sum over $n \geq 2$.
Note: the solution to the above differential equation is $b(x) = e^{x+x^2/2}$.

Q2: How many ways are there of colouring n INDISTINGUISHABLE balls using n colours, so that no colour is used 4 times or more?
[Hint: let A_i={colourings in which colour i is used at least 4 times}.]

Q3: How many pairs of permutations π_1, π_2 of $[n]$ are there for which there is no i such that $\pi_1(i) = \pi_2(i) = i$?