Discrete Mathematics 21-228: Spring 2002
Homework.
HW2 due Wednesday 1/30/2002

Q1: Show by induction that

$$\binom{n}{0}-\binom{n}{1}+\binom{n}{2}+\cdots+(-1)^k\binom{n}{k}=(-1)^k\binom{n-1}{k}.$$

Q2: Show that

$$2^{n/2}\cos n\pi/4=\binom{n}{0}-\binom{n}{2}+\binom{n}{4}-\cdots$$

[Hint: Expand $(1+i)^n$ using the binomial theorem. (Here $i^2=-1$.) Note that $1+i=\sqrt{2}(\cos\pi/4+i\sin\pi/4)$ and $e^{i\theta}=\cos\theta+i\sin\theta$.]

Q3: Show that

$$1^2\binom{n}{1}^2+2^2\binom{n}{2}^2+\cdots+n^2\binom{n}{n}^2=n^2\binom{2n-2}{n-1}.$$

[Hint: relate the expression to the number of ways of choosing 2 equal sized sets $A,B\subseteq\{1,2,\ldots,n\}$ and then choosing one "special" element from each of A,B. The LHS is easy. For the RHS imagine that A,B have to be chosen from disjoint copies of $[n]$.]