21-301 Combinatorics
Homework 8
Due: Monday, November 6

1. Let the sequence of integers $M_1, M_2, M_3, \ldots,$ be defined by the recurrence relation

$$M_1 = 3 \text{ and } M_k = kM_{k-1} - k + 2 \qquad (k > 1).$$

Show by induction on k that if the edges of K_{M_k} are coloured in k colours then there is a monochromatic triangle.
(Hint: let V_i denote the set of vertices v for which edge $\{1, v\}$ has color i.)

2. Let $m = s(p-1)+1$ and $n = s^m(q-1)+1$. Show that every s-coloring of the edges of $K_{m,n}$ using s colors contains a monochromatic copy of $K_{p,q}$.
(Hint: let X, Y be the two parts of the bipartition in $K_{m,n}$. Begin by showing that there must be q vertices $Q \subseteq Y$ such that $c(x,y) = c(x,y')$ for all $x \in X$ if $y, y' \in Q$.)

3. Prove that for $n = 2m$ sufficiently large, every 2-coloring of the edges of $K_{n,n}$ contains a monochromatic copy of $K_{p,p}$.
(Hint: suppose that $X = \{x_1, x_2, \ldots, x_n\}, Y = \{y_1, y_2, \ldots, y_n\}$ are the two parts of the bipartition in $K_{n,n}$. Consider the 2-coloring of K_m induced by the colors of the edges (x_i, y_{j+m}) for $i, j \le m$.)