21-301 Combinatorics
Homework 7
Due: Monday, October 30

1. Use the pigeon-hole principle to show that for every integer $k \geq 1$ and prime $p \neq 2, 5$ there exists a power of p that ends with $000 \cdots 0001$ (k 0's).
   (Hint: consider the sequence $p^\ell \mod 10^{k+1}, \ell = 1, 2, \ldots,.$)

2. Suppose that we two-color the edges of K_6 with Red and Blue. Show that there are at least two monochromatic triangles.

3. Show that $r(C_4, C_4) = 6$ where C_4 denotes a cycle of length four.