21-301 Combinatorics
Homework 10
Due: Wednesday, November 29

1. Find the set of P-positions for the take-away games with subtraction sets

   (a) $S = \{1, 3, 7\}$.

   (b) $S = \{1, 4, 6\}$.

   Suppose now that there are two piles and the rules for each pile are as above. Now find the P positions for the two pile game where in one pile S is as in (a) and the other pile is as in (b).

2. Consider the following game: There is a pile of n chips. A move consists of removing any *proper* factor of n chips from the pile. (For the purposes of this question, a proper factor of n, is any factor, including 1, that is strictly less than n.)The player to leave a pile with one chip wins. Determine the N and P positions and a winning strategy from an N position.

3. In a take-away game, the set S of the possible numbers of chips to remove is finite. Show that the Sprague-Grundy numbers satisfy $g(n) \leq |S|$ where n is the number of chips remaining.