

21-301 Combinatorics

Homework 1

Due: Wednesday, September 6

1. How many sequences $(a_1, a_2, \dots, a_m) \in [n]^m$ satisfy $a_1 < a_2 < \dots < a_m$? How many satisfy $a_1 \leq a_2 \leq \dots \leq a_m$?
2. For $S \subseteq [n]$, a run in S is a maximal set of consecutive integers in S . For example, $\{1, 2, 4, 5, 6, 9\}$ is a subset of $[9]$ with three runs; every element is in one run.
How many subsets of $[n]$ have exactly k runs?
3. Let π be a permutation of $[n]$. Show that

$$\sum_{i=1}^n |i - \pi(i)| \leq \left\lfloor \frac{n^2}{2} \right\rfloor.$$