

21-301 Combinatorics  
Homework 7  
Due: Wednesday, November 3

1. How many ways are there to arrange 2 M's, 4 A's, 5 T's and 6 H's under the condition that any arrangement and its reversal are to be considered the same.
2. Find the set of  $P$ -positions for the take-away games with subtraction sets
  - (a)  $S = \{1, 3, 7\}$ .
  - (b)  $S = \{1, 4, 6\}$ .

Suppose now that there are two piles and the rules for each pile are as above. Now find the  $P$  positions for the two pile game where in one pile  $S$  is as in (a) and the other pile is as in (b).

3. In a take-away game, the set  $S$  of the possible numbers of chips to remove is finite. Show that the Sprague-Grundy numbers satisfy  $g(n) \leq |S|$  where  $n$  is the number of chips remaining.