21-301 Combinatorics
Homework 2
Due: Wednesday, September 15

1. (a) How many strings of length n consisting of 0's and 1's have no two consecutive 1's?

   (b) How many strings of length n consisting of 0's and 1's have no three consecutive 1's and no three consecutive 0's?
   [Hint: reduce the question to (a).]

2. Find a_n if
$$a_n = 6a_{n-1} + 7a_{n-2}, a_0 = 2, a_1 = 10.$$

3. Let $S_0 = 1$ and S_n denote the number of ways that $2n$ people sitting in a cycle can shake hands without crossing arms?

   (a) Prove that $S_n = \sum_{i=1}^{n} S_{i-1}S_{n-i}$.
   (b) Deduce that $S_n = \frac{1}{n+1}\binom{2n}{n}$.

Figure 1: All handshake configurations for 8 people