

21-301 Combinatorics

Homework 1

Due: Wednesday, September 8

1. How many sequences $(a_1, a_2, \dots, a_m) \in [n]^m$ satisfy $a_1 < a_2 < \dots < a_m$? How many satisfy $a_1 \leq a_2 \leq \dots \leq a_m$?
2. Suppose that a round table has $3n$ labelled seats. Suppose that n families arrive consisting of man/woman/child. They are to be seated round the table in triples: adult,child,adult. How many ways of seating the guests are there so that no family sits together as a complete triple of adult,child,adult.
3. Suppose that we have $2n$ distinguishable balls. Suppose that there are n colors and that we have 2 balls of each color. How many ways are there of placing the balls into n distinguishable boxes, two balls per box, so that there are exactly k boxes containing balls of the same color?