Review Problems for Test 3

1. Determine the intervals on which f is increasing and the intervals on which f is decreasing. Also, find all relative maxima and all relative minima for f.

   (a) $f(x) = x^2 e^{2x}$
   (b) $f(x) = x^4 - 18x^2 + 1$
   (c) $f(x) = 9x^{2/3} - 2x^3 + 1$

2. Determine the intervals on which f is concave up and the intervals on which f is concave down. Also find all inflection points for f.

   (a) $f(x) = x^4 - 2x^3 + x - 1$
   (b) $f(x) = x^{1/3} + \frac{1}{9}x^2$

3. Find the horizontal and vertical asymptotes for each of the following.

   (a) $y = \dfrac{x}{\sqrt{4x^2+1}}$ (b) $y = \dfrac{4x^4 - 2x}{1 + x^2 + 5x^4}$

   (c) $y = \dfrac{x^3}{x^5+1}$

4. Give a careful sketch of the graph of $y = f(x)$. In particular, determine the intervals on which f is increasing, decreasing, concave up, concave down. Find all relative maxima, relative minima, and inflection points. Also, find all horizontal asymptotes and all vertical asymptotes.

   (a) $f(x) = 2x^3 - 3x^2 + 1$
   (b) $f(x) = \dfrac{x}{(x-2)^2}$ (You may take it for granted that
   $$f'(x) = \frac{-(x+2)}{(x-2)^3},\ f''(x) = \frac{2x+8}{(x-2)^4} \text{ in part b.)}$$
   (c) $f(x) = \dfrac{x^2+1}{x^2-1}$ (You may take it for granted that
   $$f'(x) = \frac{-4x}{(x^2-1)^2}, f''(x) = \frac{12x^2+4}{(x^2-1)^3} \text{ in part c.)}$$

5. Suppose that the demand for a monopolist's product satisfies

$$p = \frac{1000}{10+q}$$

where p is the price per unit and q is the number of units. Suppose further that the average cost of production is given by

$$\bar{c} = 1 + \frac{20}{q}.$$

Find the level of production and the price that maximize profit.

6. A running track is to built with two parallel straight pieces having semicircular caps at each end. (See diagram below.) The perimeter of the track is to be 1000 meters. Find the dimensions that will maximize the area of the rectangular field inside.

7. A swimmer is in the ocean 100 meters from a straight shoreline. A person in distress is located on the shoreline 300 meters from the point of the shoreline that is closest to the swimmer. If the swimmer can swim at 3 meters per second and run at 5 meters per second, where should the swimmer land on the shore in order to reach the distressed person as soon as possible.

8. A cylindrical can with an open top is to have a volume of 24π in^3. The material used to make the base costs \$3/in^2 while the material for the lateral surface costs \$1/in^2. Assuming no waste in construction, find the dimensions of the can so as to minimize the total cost of materials.

9. A person wishes to enclose a rectangular plot of land with a fence. The plot is to be divided into two pieces by a fence parallel to the sides, as shown below. The area of the smaller piece is to be 600 ft.^{2}, while the area of the large piece is to be 1200 ft.^{2}. Find the least number of feet of fence that will suffice.

10. Show that the equation

$$e^x + x - 2 = 0$$

has exactly one real root.

11. Let α, β be given real numbers. Show that the equation

$$x^6 + \alpha x + \beta = 0$$

has at most 2 real roots.

12. Find f.

(a) $f'(x) = \sqrt{x} + \frac{1}{\sqrt{x}}, \ \ f(1) = 1.$

(b) $f'''(x) = e^{-2x}.$

(c) $f'(x) = \sin 3x - 3\cos 2x.$

(d) $f'(x) = 3x^4 - 2x^3 + x - 1.$

(e) $f''(x) = x(1-x), \quad f(0) = 0, \ f(1) = 0.$