21-259 **Calculus in 3-D** **Spring 2003**

Formula Sheet for Test 3

Cylindrical Coordinates:

$$x = r\ cos\theta,\quad y = r\sin\theta,\quad z = z,\quad r = \sqrt{x^2+y^2},\quad dV = r\ dzdrd\theta$$

Spherical Coordinates:

$$x = \rho\sin\varphi\cos\theta,\ y = \rho\sin\varphi\sin\theta,\ z = \rho\cos\ \varphi,\quad \rho = \sqrt{x^2+y^2+z^2},\ dV = \rho^2\sin\varphi\ d\rho d\varphi d\theta$$

Green's Theorem:

$$\int_C P(x,y)dx + Q(x,y)dy = \int_D\int\left(\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y}\right)dA$$

Here C is a simple closed curve in the $x-y$ plane with counterclockwise orientation and D is the region enclosed by C.

Surfaces Described by $\vec{r}(u,v)$:

$$dS = \left|\frac{\partial\vec{r}}{\partial u}\times\frac{\partial\vec{r}}{\partial v}\right|dudv,\qquad \vec{n}dS = \pm\frac{\partial\vec{r}}{\partial u}\times\frac{\partial\vec{r}}{\partial v}dudv$$

Surfaces Described by $z = f(x,y)$:

$$dS = \sqrt{1+f_x^2+f_y^2}\ dydx,\qquad \vec{n}dS = \pm\left(f_x\vec{i}+f_y\vec{j}-\vec{k}\right)dydx$$

Sphere of Radius $a > 0$ Centered at $(0,0,0)$:

$$\vec{r}(\varphi,\theta) = (a\sin\varphi\cos\theta)\vec{i} + (a\sin\varphi\sin\theta)\vec{j} + (a\cos\varphi)\vec{k}\quad dS = a^2\sin\varphi d\varphi d\theta$$