Quiz 9

Problem. Find two positive intergers where the sum of the first and four times the second is 1000 and the product is as large as possible.

Solution. The algebra is

$$P = xy$$
$$x + 4y = 1000$$

Thus, using the second equation, we can eliminate a variable from the first. Thus we are left with maximizing

$$P = (1000 - 4y)y = 1000y - 4y^2$$

Taking the first derivative, we have

$$P' = 1000 - 8y$$

Setting this equal to 0, we see that $y = \frac{1000}{8} = 125$. The x corresponding to this y is $x = 500$. We now only need to show this is a local max. To see that, we can observe that the parabola $1000y - 4y^2$ is opening down, which means it must be a max. A more systematic way, however, is to take the second derivative

$$P'' = -8$$

Thus, this is concave down (looks like a frown) and must be a maximum.