Quiz 2

Problem.

1. Use the definition of the derivative to find $\frac{d}{dx}f(x)$ given $f(x) = \sqrt{1+2x}$.
2. Find the equation of line tangent to $f(x) = x^2 + x$ at $x = 2$

Solution.

1.

$$\begin{aligned}
\frac{d}{dx}f(x) = \lim_{h\to 0}\left(\frac{\sqrt{1+2(x+h)}-\sqrt{1+2x}}{h}\right) &= \lim_{h\to 0}\left(\frac{\sqrt{1+2(x+h)}-\sqrt{1+2x}}{h}\cdot\frac{\sqrt{1+2(x+h)}+\sqrt{1+2x}}{\sqrt{1+2(x+h)}+\sqrt{1+2x}}\right)\\
&= \lim_{h\to 0}\left(\frac{1+2(x+h)-1-2x}{h(\sqrt{1+2(x+h)}+\sqrt{1+2x})}\right)\\
&= \lim_{h\to 0}\left(\frac{1+2x+2h-1-2x}{h(\sqrt{1+2(x+h)}+\sqrt{1+2x})}\right)\\
&= \lim_{h\to 0}\left(\frac{2h}{h(\sqrt{1+2(x+h)}+\sqrt{1+2x})}\right)\\
&= \lim_{h\to 0}\left(\frac{2}{\sqrt{1+2(x+h)}+\sqrt{1+2x}}\right)\\
&= \frac{2}{\sqrt{1+2x}+\sqrt{1+2x}}\\
&= \frac{1}{\sqrt{1+2x}}
\end{aligned}$$

2. First we need to take the derivative. We will use the power rule and we determine that

$$f'(x) = 2x + 1$$

Plugging in $x = 2$ we see that $f'(2) = 5$. The function value at 2 is given by $f(2) = (2)^2 + 2 = 6$. Recall point slope form of a line, ie. if m is the slope, and (x_1, y_1) is a point, then the equation of the line is given by

$$y - y_1 = m(x - x_1)$$

Therefore, we can see the equation of the tangent line is

$$y - 6 = 5(x - 2)$$

Or, written a bit differently

$$y = 5x - 4$$