

**Problem.**

1.  $\int \left(x + \frac{1}{x}\right) dx$

2.  $\int \left(x^3 + 3x^2 + 2x + \frac{1}{3}\right) dx$

**Bonus.**  $\int \sin\left(\frac{x}{2}\right) \cos\left(\frac{x}{2}\right) dx$

**Solution.**

1.

$$\begin{aligned}\int \left(x + \frac{1}{x}\right) dx &= \left(\int x dx\right) + \left(\int \frac{1}{x} dx\right) \\ &= \frac{x^2}{2} + \ln|x| + C\end{aligned}$$

2.

$$\begin{aligned}\int \left(x^3 + 3x^2 + 2x + \frac{1}{3}\right) dx &= \left(\int x^3 dx\right) + \left(3 \int x^2 dx\right) + \left(2 \int x dx\right) + \left(\frac{1}{3} \int dx\right) \\ &= \frac{x^4}{4} + x^3 + x^2 + \frac{x}{3} + C\end{aligned}$$

**Bonus.** Recall,  $\sin(2\theta) = 2 \sin(\theta) \cos(\theta)$ . So,

$$\begin{aligned}\int \sin\left(\frac{x}{2}\right) \cos\left(\frac{x}{2}\right) dx &= \int \frac{\sin(x)}{2} \\ &= \frac{1}{2} \int \sin(x) \\ &= -\frac{\cos(x)}{2}\end{aligned}$$