Lecture: MWF 10:10 – 11:00 am, Doherty Hall 2105

Lecturer: Tomasz Tkocz, Wean Hall 7206, `ttkocz@math.cmu.edu`

TA: Shimaa Elesaely, `selesael@andrew.cmu.edu`

Office Hours: ... or by email appointment

Course website: `Canvas` and `http://math.cmu.edu/~ttkocz`

Course description: This course is a rigorous introduction to probability theory, starting from the definition of a probability space with the main objectives being the law of large numbers, the central limit theorem, elements of martingale theory, concentration inequalities, large deviations and Markov chains (time permitting).

Prerequisites: basics of linear algebra; basics of complex analysis; measure theory; an undergraduate course in probability theory is not required but can be helpful to develop intuition

Literature:

- Rosenthal, J., A first look at rigorous probability theory. *World Scientific Publishing*, 2006.
- Williams, D., Probability with martingales. *Cambridge University Press*, 1991.
- Billingsley, P., Probability and measure. *John Wiley & Sons*, 1979.
- Durrett, R., *Probability: Theory and Examples.* Available online on the author's website `https://services.math.duke.edu/∼rtd/PTE/PTEv5a.pdf`
- Kallenberg, O., Foundations of modern probability, *Springer New York*, 2002.
- Shiryaev, A. N., Probability. *Graduate Texts in Mathematics, 95. Springer-Verlag*, 1996.

Course content: probability spaces, random variables, expectation, independence, Kolmogorov's 0-1 law, Borel-Cantelli lemmas, weak and strong laws of large numbers, Fourier analytic techniques: characteristic functions, Lindeberg's central limit theorem, the Berry-Esseen theorem via Lindeberg's argument as well as Stein's method, an example of local limit theorems for integer-valued random variables, filtration, martingales, stopping times, upcrossing inequality and martingale convergence theorems, backward martingales, maximal inequalities, applications of martingales, large deviations, rate functions, Cramer's Theorem, Bernstein's, Hoeffding's, Azuma's inequalities, Chernoff bounds.

Learning objectives:

- understanding the role of a probability space and basic distributions in building appropriate probabilistic models
- understanding several important basic probabilistic techniques with applications in e.g. analysis and combinatorics
- understanding several important probabilistic phenomena related to independence: law of large numbers and central limit theorem
- understanding probabilistic aspects of martingales (fair games) and their applicability and ubiquity

Course format: This is an in person class. You are expected to fully participate in class, viz. please ask and answer questions, initiate or participate in discussions. We follow rather closely the Stein-Shakarchi textbook.

Homework: There will be about 12 homework assignments during the semester.

Late submissions will not be accepted, but the lowest homework score will not count towards the final grade. Plagiarism is not tolerated. Collaboration on homework is allowed, but has to be acknowledged in writing and the solutions must be written on your own, at least one tea break after the collaboration ended.

The assignments will be administered via Gradescope. Only high quality pdf-scans of hand-written solutions will be accepted (consider apps like Dropbox, or Notes on iOS to produce them), or use LaTeX.

Exams: There will be 4 in-class tests throughout the semester (based on the practice problems and the lecture material). *No* final exam, *but* suggested grades will be out before the end of the semester and you can request an oral final examination to improve your grade. Plagiarism and cheating are not tolerated.

Grades: The midterm grade will be based solely on homework. The final grade will be based on homework and tests, computed as a weighted average:

50% **Homework** + 50% **Tests**

Rough guide on "score" → "grade" map: `https://en.wikipedia.org/wiki/Academic_grading_in_the_United_States` (but the grades will be "curved" if needed)

Young man, in mathematics you don't understand things. You just get used to them.

–John Von Neumann to Felix Smith