Spring 2022 **Complex Analysis 21-623** **CMU**

Lecture: MWF 9:05 am – 9:55 pm, Porter Hall A22

Lecturer: Tomasz Tkocz, Wean Hall 7206, `ttkocz@math.cmu.edu`

TA: Wei Dai, `wdei2@andrew.cmu.edu`

Office Hours: ... or by email appointment

Course website: `Canvas` and/or `http://math.cmu.edu/~ttkocz`

Course description: This course is a rigorous introduction to complex analysis, starting from the notion of complex differentiation, entering a marvelous world, full of wonderful insights, where functions differentiable once are automatically indefinitely differentiable. The highlights include slick proofs of the fundamental theorem of algebra, the Riemann zeta function central in the theory of prime numbers, the Fourier transform, conformal mappings and (un)expected applications of these in other areas of mathematics.

Prerequisites: complex numbers, certain maturity with real analysis (for instance within the scope of a solid undergraduate class such as Principles of Real Analysis I and II)

Literature:

- Ahlfors, L., Complex Analysis : An Introduction to The Theory of Analytic Functions of One Complex Variable. *McGraw-Hill*, 1979.
- Stein, E., Shakarchi, R., Complex Analysis. *Princeton University Press*, 2003.
- Newman, D. J., Analytic Number Theory. *Springer*, 1998.

Course content: functions on the complex plane, complex derivative, the Cauchy-Riemann equations, integration along curves, Goursat's theorem, Cauchy's theorem and integral formulas, Morera's theorem, Schwarz reflection principle, zeros and poles, the residue formula, singularities and meromorphic functions, the argument principle and applications, Rouché's theorem, the open mapping theorem, the maximum principle, the Fourier transform, Paley-Wiener type theorems, Jensen's formula, Hadamard's factorisation theorem, the gamma and zeta functions, conformal mappings, the prime number theorem, elliptic functions (time permitting)

Learning objectives: Students should
- gain understanding of basic properties of analytic functions
- advance their insight into the interplay between algebraic, geometric and analytic ideas
- develop an improved ability and use the methods and results of complex analysis, with applications in other areas, particularly algebra, number theory, combinatorics, geometry and probability

Course format: This is an in person class. You are expected to fully participate in class, viz. please ask and answer questions, initiate or participate in discussions. We follow rather closely the Stein-Shakarchi textbook.

Homework: There will be about 12 homework assignments during the semester.

Late submissions will not be accepted, but the lowest homework score will not count towards the final grade. Plagiarism is not tolerated. Collaboration on homework is allowed, but has to be acknowledged in writing and the solutions must be written on your own, at least one tea break after the collaboration ended.

The assignments will be administered via Gradescope. Only high quality pdf-scans of hand-written solutions will be accepted (consider apps like Dropbox, or Notes on iOS to produce them), or use LaTeX.

Exams: There will be 4 in-class tests throughout the semester (based on the practice problems and the lecture material). *No* final exam, *but* suggested grades will be out before the end of the semester and you can request an oral final examination to improve your grade. Plagiarism and cheating are not tolerated.

Grades: The midterm grade will be based solely on homework. The final grade will be based on homework and tests, computed as a weighted average:

50% **Homework** + 50% **Tests**

Rough guide on "score" → "grade" map: `https://en.wikipedia.org/wiki/Academic_grading_in_the_United_States` (but the grades will be "curved" if needed)

The sweeping development of mathematics during the last two centuries is due in large part to the introduction of complex numbers; paradoxically, this is based on the seemingly absurd notion that there are numbers whose squares are negative.

–E. Borel, 1952