Fall 202 **Linear Algebra 21-341** **CMU**

Lecture: MWF 4:00 – 4:50 pm (Pittsburgh time), Doherty Hall 2315 and Zoom

Lecturer: Tomasz Tkocz, `ttkocz@math.cmu.edu`

Office Hours: Zoom (time-slots set in the first week of classes/email appointment)

Course website: `Canvas` and `http://math.cmu.edu/~ttkocz/teaching_2021.php`

Course description: Historically, linear algebra grew out of solving systems of linear equations and investigating the structure of their solution-sets (Cramer's rule goes back to 1750). Modern linear algebra concerns vector spaces (first introduced by Peano in 1888) and linear maps. It is central to almost all areas of mathematics and applications. This course systematically introduces (from scratch) the core material of linear algebra that every mathematician should know, supplemented with often unexpected applications in other areas, particularly combinatorics. It is proof based, intended for advanced undergraduate students with some level of mathematical maturity, who ideally have been exposed to basic (computational) aspects of linear algebra in $\mathbb{R}^n$.

Course content: fields, vector spaces, systems of linear equations, linear operators and matrices, duality, determinants, tensor product and exterior algebra, polynomials, invariant subspaces, inner-product spaces, spectral theory, discrete Fourier expansion, structure theorems, quadratic forms, Perron-Frobenius theory (time permitting)

Literature:

- Y & Y Katznelson, *A (Terse) Introduction to Linear Algebra.* Concise, covers most of the material of this course (not recommended for self-study)
- J. Matoušek, *Thirty-three Miniatures: Mathematical and Algorithmic Applications of Linear Algebra* (applications!)
- S. Lang, *Linear Algebra.* (recommended for self-study)
- K. M. Hoffman, R. Kuze, *Linear Algebra.* (recommended for self-study)
- S. Axler, *Linear Algebra Done Right.* (available on-line via CMU library)
- S. Treil, *Linear Algebra Done Wrong.* (available on-line `https://www.math.brown.edu/~treil/papers/LADW/LADW.html`)

Learning objectives: Students should

- gain understanding of basic definitions and theorems concerning the core topics of linear algebra listed above
- develop an improved ability and use the methods and results of linear algebra, with applications in other areas, particularly analysis, geometry and combinatorics

Course format: There are three lectures per week. Lecture notes will be regularly uploaded on the course website. There are weekly assignments. There is one midterm and the final exam. This is an in person/remote class. You choose your mode of participation and are free to change it at any point. You are expected to fully participate in class, viz. please ask and answer questions, initiate or participate in discussions. If you attend remotely, you are very much encouraged to keep you camera on to facilitate interactions and help me judge pace/understanding/etc. If you attend in person, you must wear your face-mask at all times.

Recodrings: Lectures will be recorded and the videos will be readily made available on-line. You must not distribute, share or post the recordings.

Homework: This is the essential part of the learning process in this course. Simply listening in class or reading texts is not sufficient. Understanding mathematics requires practice. The course will be fast-paced, therefore weekly assignments will help you study systematically, without gaps in comprehending the material.

Late submissions will not be accepted, but the two lowest homework scores will not count towards the final grade. You may lose points for poor presentation. Please write neatly and provide complete solutions, all explanations and arguments, not just answers. Plagiarism is not tolerated. Collaboration on homework is allowed, but has to be acknowledged in writing and the solutions must be written on your own, at least one tea break after the collaboration ended. Collaboration on take-home tests is forbidden.

The assignments will be administered via Gradescope. Only high quality pdf-scans of hand-written solutions will be accepted (consider apps like Dropbox, or Notes on iOS to produce them), or use LaTeX.

Exams: There will be one take-home midterm. There will be a final take-home exam. Both one-week long. Exam questions will mainly be homework-style problems.

Grades: The midterm grade will be based solely on the first midterm exam. The final grade will be based on homework, midterms and the final exam, computed as a weighted average:

$$50\% \textbf{ Homework } + 20\% \textbf{ Midterm } + 30\% \textbf{ Final exam}$$

Rough guide on "score" $\to$ "grade" map: `https://en.wikipedia.org/wiki/Academic_grading_in_the_United_States` (but the grades will be "curved" if needed)

Tips: Expect to budget at least 10 hours per week of work (outside lecture) for this course. Solving problems, reworking your notes, understanding all the definitions, examples, theorems, their assumptions, etc. will take time, but being systematic and consistent will pay off. Come to class prepared, zealous and eager to learn. You will be surprised how much of a difference it will make, if you spend say 15 min revising the recent material before coming to class. Start working on homework early to give yourself enough time to think and seek my help if you get stuck. I am available by email.

The Matrix is everywhere.

–Morpheus

It is my experience that proofs involving matrices can be shortened by 50% if one throws the matrices out.

–Emil Artin

Come, every frustum longs to be a cone,
And every vector dreams of matrices.

–From The Cyberaid by Stanisław Lem