Spring 2020 **Probability 21-721** **CMU**

Lecture: MWF 10:30 – 11:20 am, Wean Hall 8220

Lecturer: Tomasz Tkocz, Wean Hall 7206, `ttkocz@math.cmu.edu`

Office Hours: 9:00 – 10:30 am Mon (tentatively) or by email appointment

Course website: http://www.math.cmu.edu/∼ttkocz/teaching_current.php

Course description: This course is a rigorous introduction to probability theory, starting from the definition of a probability space with the main objectives being the law of large numbers, the central limit theorem, elements of martingale theory, concentration inequalities, large deviations and Markov chains (time permitting).

Prerequisites: basics of linear algebra; basics of complex analysis; measure theory; an undergraduate course in probability theory is not required but can be helpful to develop intuition

Literature:

- Rosenthal, J., A first look at rigorous probability theory. *World Scientific Publishing*, 2006.
- Williams, D., Probability with martingales. *Cambridge University Press*, 1991.
- Billingsley, P., Probability and measure. *John Wiley & Sons*, 1979.
- Durrett, R., *Probability: Theory and Examples.* Available online on the author's website `https://services.math.duke.edu/∼rtd/PTE/PTEv5a.pdf`
- Kallenberg, O., Foundations of modern probability, *Springer New York*, 2002.
- Shiryaev, A. N., Probability. *Graduate Texts in Mathematics, 95. Springer-Verlag*, 1996.

Course content: probability spaces, random variables, expectation, independence, Kolmogorov's 0-1 law, Borel-Cantelli lemmas, weak and strong laws of large numbers, characteristic functions, Lindeberg's central limit theorem, the Berry-Esseen theorem via Stein's method, an example of local limit theorems, filtration, martingales, stopping times, upcrossing inequality and martingale convergence theorems, backward martingales, maximal inequalities, applications of martingales, large deviations, rate functions, Cramer's Theorem, Bernstein's, Hoeffding's, Azuma's inequalities, Chernoff bounds.

Learning objectives:

- understanding the role of a probability space and basic distributions in building appropriate probabilistic models
- understanding several important basic probabilistic techniques with applications in e.g. analysis and combinatorics
- understanding several important probabilistic phenomena related to independence: law of large numbers and central limit theorem
- understanding probabilistic aspects of martingales (fair games) and their applicability and ubiquity

Course oragnisation: There are three lectures per week. We follow rather closely two basic textbooks by Rosenthal and Williams, with some topics expanded. Comprehensive classical positions such as Billingsley, Durrett or Shiryaev are also recommended. Lecture notes will be regularly uploaded on the course website. There are weekly assignments. There is a midterm and the final exam.

Homework: This is the essential part of the learning process in this course. Simply listening in class or reading texts is not sufficient. Understanding mathematics requires practice. The course will be fast-paced, therefore weekly assignments will help you study systematically, without gaps in comprehending the material.

Weekly assignments will be posted on the course website at least one week before the due date. The assignments will be due on Wednesdays, collected in class, *before* the lecture begins. Late submissions will not be accepted. At the end of the course, the two lowest homework scores will not count towards the final grade. Collaboration is encouraged, but plagiarism is not tolerated (that is, you should write solutions on your own).

Exams: There will be a midterm exam taken during class. There will be a take-home final exam covering all the material (released before the last week of classes). Exam questions will be based largely on homework problems.

Grades: The midterm grade will be based solely on the midterm exam. The final grade will be based on homework, the midterm and the final exam, computed as a weighted average: 30% Homework + 30% Midterms + 40% Final exam