Probability 21-325 **Homework 2 (due 30th Jan)** **TT**

1. Urn I contains 4 white and 3 black balls and Urn II contains 3 white and 7 black balls. An urn is selected at random and a ball is picked from it. What is the probability that this ball is black? If this ball is white, what is the probability that Urn I was selected?

2. Two fair dice are thrown. Let A be the event that the first shows an odd number, B be the event that the second shows an even number and C be the event that either both are odd or both are even. Are the events A, B and C independent? Are they pairwise independent?

3. A single card is removed from a standard deck of 52 cards. From the remainder we draw two cards at random and find that they are both spades. What is the probability that the first card removed was also a spade?

4. There are n invitation cards with the names of n different people and n envelopes with their names. We put the cards at random into the envelopes, one card per envelope. What is the chance that not a single invitation landed in the correct envelope? What is the limit of this probability as n goes to infinity?

5. Prove that for positive integers m, n and two positive numbers p, q satisfying $p+q=1$ we have

$$(1-p^n)^m + (1-q^m)^n \geq 1.$$

6.* Prove that for a positive integer n and two real numbers x, y satisfying $x+y=1$ we have

$$x^{n+1}\sum_{k=0}^{n}\binom{n+k}{k}y^k + y^{n+1}\sum_{k=0}^{n}\binom{n+k}{k}x^k = 1.$$

7. There are n balls in an urn. They are labelled $1, 2, \ldots, n$. We randomly pick k balls (without replacement), one by one. Find the probability that the label on the kth ball is larger than on all previously picked balls.

8. A bamboo stick is randomly broken at two points. What is the probability that from the three pieces we get we can assemble a triangle?

9. A needle of length ℓ is dropped on a plane ruled with parallel lines which are d apart. Assume that $\ell < d$. What is the probability that the needle will cross a line?

10.* Show that an infinite σ-field is uncountable.