Fall 2018 **An introduction to asymptotic convex geometry** **CMU**

Lecture: Mon Wed Fri 1:30 – 2:20 pm, Wean Hall 7201

Lecturer: Tomasz Tkocz, Wean Hall 7206, `ttkocz@math.cmu.edu`

Office Hours: Wed 2:30 – 4:30 pm

Course website: `https://www.mimuw.edu.pl/~tkocz/teaching_current.php`

Course description: This course is an introduction to asymptotic convex geometry which is the study of convex sets in high dimensions and the dimension dependence of their various parameters. We shall try to understand from an analytical as well as geometric point of view, how the convexity of a set forces most of its volume to be distributed in a canonical way. To this end, we shall introduce a number of classical tools, techniques and theorems (e.g. the Brunn-Minkowski inequality, isoperimetry, John's position, concentration, Dvoretzky's theorem) and then move on to recent results and open questions, mainly related to volume distribution in convex sets.

Prerequisites:

- solid background in linear algebra: spectral theorem, singular value decomposition, positive definite matrices (e.g. *21-341 Linear Algebra*)
- solid background in analysis: integration in $\mathbb{R}^n$, $L_p(\mu)$ spaces, Hölder's inequality, Jensen's inequality (e.g. *21-268* or *21-269 Vector Analysis*, *21-355 Principles of Real Analysis I*; knowledge from *21-640 Introduction to Functional Analysis*, although not crucial, can be advantageous)
- basic probability: random vectors in $\mathbb{R}^n$, Gaussian distribution, CLT (e.g. *21-325 Probability*)

Literature:

- Ball, K., An elementary introduction to modern convex geometry. Cambridge, 1997.
- Ball, K., Convex geometry and functional analysis. Handbook of the geometry of Banach spaces, Vol. I, Amsterdam, 2001.
- Artstein-Avidan, S., Giannopoulos, A., Milman, V., Asymptotic geometric analysis. Part I. Providence, RI, 2015.
- Brazitikos, S., Giannopoulos, A., Valettas, P., Vritsiou, B., Geometry of isotropic convex bodies. Providence, RI, 2014.

Course content: basic convexity, Brunn-Minkowski and Prékopa-Leindler inequalities, geometric forms of Brascamp-Lieb inequalities, isoperimetry (classical and reverse), classical positions of convex bodies, concentration of measure (spherical, Gaussian, Talagrand's inequality on the discrete cube, Borell's lemma), Dvoretzky's theorem, Kashin's theorem; further, depending on time: isotropic constant, slicing, isomorphic slicing, volume distribution in convex bodies (small ball and deviation inequalities)

Learning objectives:

- understanding the role of symmetry and convexity in various volumetric properties in high-dimensions
- understanding several important high-dimensional phenomena related to volume distribution in convex sets such as concentration of measure
- classical and modern techniques from analysis, probability and geometry motivated by problems in asymptotic theory of convex sets (tensorisation, localisation, symmetrisation, centroid bodies, Gaussian processes, etc.)

Course oragnisation: There are three 50 min lectures per week. Lecture notes will be regularly uploaded on the course website. There are 4 homework assignments.

Homework: This is the essential part of the learning process in this course. Simply listening in class or reading texts is not sufficient. Understanding mathematics requires practice. The course will be fast-paced, therefore assignments will help you study systematically, without gaps in comprehending the material.

Assignments will be posted on the course website at least two weeks before the due date. The assignments will be collected in class, *before* the lecture begins. Late submissions will not be accepted. You may lose points for poor presentation. Please write neatly and provide complete solutions, all explanations and arguments, not just answers. Plagiarism is not tolerated.

Exams: There will be a take-home exam during the official exam period.

Grades: The final grade will be based on homework (70%) and the final exam (30%).