

Please write down your name in capital letters. No resources allowed (books, notes, electronic devices, etc.)

1. We randomly put 20 identical balls into 7 labelled boxes (multiple occupancies are allowed). What is the probability that at least one box is empty?
2. A fair die is thrown 4 times. For $i = 1, \dots, 6$, let X_i be a random variable equal to 1 if the side showing “ i ” was rolled at least once and equal to 0 otherwise. What is the distribution of X_i called? Find the expectation of X_i . Let X be the number of different outcomes obtained (for example, if the outcomes are 4, 6, 6, 1, then $X = 3$). Express X in terms of the X_i . Find the expectation of X .
3. Let A_1, \dots, A_n be events such that $\mathbb{P}(A_k) = 1 - \frac{1}{2^k}$ for each $k = 1, \dots, n$. Show that the set $\bigcap_{k=1}^n A_k$ is nonempty.