1. Is the function $G(x, y) = \mathbf{1}_{\{x+y\geq 0\}}$ the distribution function of some random vector in $\mathbb{R}^2$? Explain.

2. Let X and Y be independent exponential random variables with parameters λ and μ. Show that $\min\{X, Y\}$ has the exponential distribution with parameter $\lambda + \mu$.

3. If X has the exponential distribution, show the *lack of memory* property: for every positive s and t,

$$\mathbb{P}(X > s + t | X > s) = \mathbb{P}(X > t).$$

4. Let $X_1, \ldots, X_n$ be independent exponential random variables with parameter 1. Find the distribution function of $Y_n = \max\{X_1, \ldots, X_n\}$. What is the pointwise limit of the distribution function F_n of $Y_n - \log n$? Is the limiting function a distribution function?

5. Let $X_1, X_2, \ldots$ be i.i.d. continuous random variables. Define N as the unique index such that

$$X_1 \geq X_2 \geq \ldots \geq X_{N-1} \text{ and } X_{N-1} < X_N.$$

Prove that $\mathbb{P}(N = k) = (k-1)/k!$, $k = 1, 2, \ldots$ and find $\mathbb{E}N$.

6. Find a constant C such that $f\colon \mathbb{R}^2 \to \mathbb{R}$ given as $f(x, y) = \frac{C}{(1+x^2+y^2)^{3/2}}$ is a density function. Show that both marginals have the Cauchy distribution.

7. Let $X_1, X_2, \ldots$ be independent exponential random variables with parameter 1. Show that for every n, the distribution of $X_1 + \ldots + X_n$ is Gamma(n).

8. Let (X, Y) be a random vector in $\mathbb{R}^2$ with density $f(x, y) = cxy\mathbf{1}_{0<x<y<1}$. Find c and $\mathbb{P}(X + Y < 1)$. Are X and Y independent? Find the density of $(X/Y, Y)$. Are X/Y and Y independent? What is the conditional density of X given $Y = y$?

9. Let X and Y be independent standard Gaussian random variables. Show that X/Y has the Cauchy distribution. Find $\mathbb{P}\left(X^2 + Y^2 < a\right)$ for $a > 0$ and $\mathbb{E}\sqrt{X^2 + Y^2}$.

10.* Let X be a standard Gaussian random variable and Y be an exponential random variable with parameter 1. Show that $\sqrt{2Y}X$ has the symmetric (two-sided) exponential distribution with parameter 1.