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Parabolic equations with anisotropic non-uniform degeneracy

Let $\Omega \subset \mathbb{R}^n$ be a bounded domain with Lipschitz-continuous boundary Γ and $Q = \Omega \times (0, T]$. We study the parabolic equations with anisotropic non-uniform degeneracy

$$u_t - \sum_{i=1}^{n} D_i \left(a_i(x,t,u)|D_i u|^{p_i(x,t)-2} D_i u \right) + c(x,t,u)|u|^{\sigma(x,t)-2} u = f(x,t) \quad \text{in } Q, \tag{1}$$

$$u_t - \sum_{i=1}^{n} D_i \left(a_i(x,t,u)|u|^{\gamma_i(x,t)} D_i\, u + b_i(x,t,u)|u|^{\delta_i(x,t)} u \right) + c(x,t,u)|u|^{\sigma(x,t)} u = f(x,t) \tag{2}$$

under the boundary and initial conditions

$$u = 0 \text{ on } \Gamma, \qquad u(x,0) = u_0(x) \text{ in } \Omega. \tag{3}$$

The coefficients γ_i, δ_i, a_i, b_i, c, p_i and σ are given functions of their arguments. Such equations emerge from the mathematical modelling of electrorheological fluids, fluids with temperature-dependent viscosity, the processes of filtration in inhomogenenous anisotropic media. It is assumed that the coefficients a_i, c are bounded, $a_i > 0$, $c \geq 0$, and the exponents $p_i(x,t) > 1$, $\sigma(x,t) > 1$ are continuous in Q with a logarithmic module of continuity.

We prove the existence and uniqueness of weak solutions of problems (**??**), (**??**) and (**??**), (**??**) and study the localization (vanishing) properties of weak solutions and the effect of finite time stabilization of solutions to a stationary profile. The study of the localization properties is performed with the method of local energy estimates [**?**]. The detailed proofs can be found in [**?**, **?**, **?**, **?**]

References

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