Submacroscopically Stble Equilibria of Bodies Undergoing Disarrangements and Dissipation

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Abstract: Elasticity is viewed here as a starting point in the description of inelastic behavior. The two-scale geometry provided by structured deformations and a field theory of elastic bodies undergoing disarrangements (non-smooth geometrical changes) and dissipation are used to formulate and illustrate a concent of "submacroscopically stable configuration." A body in a submacroscopically stable equilibrium configuration resists additional submacroscopic geometrical changes such as the occurrence of microslips, the formation of microvoids, and the appearance of localized distortions that, together, leave the macroscopic configuration of the body unchanged. Submacroscopically stable configurations represent energetically preferred phases for the bodies in equilibrium, and a procedure is described here for determining the submacroscopically stable equilibra of a body. The procedure is carried out in detail here for two classes of bodies that may undergo disarrangements and experience internal dissipation. One class is characterized by its bi-quadratic free energy response function, and the requirement of submacroscopic stability reduces from five to one the number of phases available to a body that is in equilibrium under mixed boundary conditions. For subclass of these bi-quadratic free energy functions, the distinguished phase determined through this procedure has the same free energy response as one derived for nematic elastomers via statistical calculations. Boundary-value problems for the macroscopic deformation corresponding to a submacroscopically stable equilibrium in a body of this class are formulated. A second class of bodies, the "near-sighted fluids," has both a prolate and a spherical phase

that may occur in equilibrium; the submacroscopically stable equilibria of a near-sighted fluid must be stress-free, without regard to the particular phase that appears. In all considerationsin this article, the term “equilibrium” is synonymous wiith satisfaction of balance of forces and moments in a given environment.