Divorcing Pressure from Viscosity in Incompressible Navier-Stokes Dynamics

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Abstract: We show that in bounded domains with no-slip boundary conditions, the Navier-Stokes pressure can be determined in a such way that it is strictly dominated by viscosity. As a consequence, in a general domain we can treat the Navier-Stokes equations as perturbed vector diffusion equation, instead of as a perturbed Stokes system. We illustrate the advantages of this view in a number of ways. In particular, we provide simple proofs of (i) local-in-time existence and uniqueness of strong solutions for an unconsrained formulation of the Navier-Stokes equations, and (ii) the unconditional stability and convergence of difference schemes that are implicit only in viscosity and explicit in both pressure and convection terms, requiring no solution of stationary Stokes systems or inf-sup conditions.