COMPOSITES OF HYPERELASTIC MATERIALS WITH PRESCRIBED LAVRENTIEV GAP FUNCTIONS

Dedicated to Walter Noll and Jerry Ericksen in Honor of Their 80th Birthdays

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In 1985, J. M. Ball and V. J. Mizel raised the question of whether there exist nonlinearly elastic materials possessing a physically natural stored energy density [one which is independent of an observer's coordinate frame (objective) and is invariant under a group of linear transformations (isotropic)] as well as physically reasonable boundary value problems for such materials, such that the infimum of the total stored energy for continuous deformations of the material meeting the boundary conditions (admissible deformations) and belonging to a Sobolev space W^{1,p_2} for some $p_2 > 1$ is strictly greater than its infimum for those admissible deformations which belong to a Sobolev space W^{1,p_1} with $p_1 < p_2$, despite the density of the former Sobolev space in the latter. The question was motivated by M. Lavrentiev's 1926 demonstration of such a gap for 1-dimensional variational boundary value problems on a bounded interval whose smooth integrand satisfied the conditions of Tonelli's existence theorem - as well as improved versions developed in the 1980's. Thereafter, M. Foss demonstrated in 2000 that there are (nonphysical) model problems in which the infimum over $W^{1,p}$ varies continuously with p. The positive (2-dimensional) resolution of the 1985 Ball/Mizel question was achieved by Foss, Hrusa and Mizel in 2003.