Singular Perturbation Models in Phase Transitions for Second Order Materials

M. Chermisi
Department of Mathematical Sciences
Carnegie Mellon University
Pittsburgh, PA, U.S.A.
chermisi@andrew.cmu.edu

G. Dal Maso
SISSA, Trieste, Ital
dalmaso@sissa.it

I. Fonseca
Department of Mathematical Sciences
Carnegie Mellon University
Pittsburgh, PA, U.S.A.
fonseca@andrew.cmu.edu

G. Leoni
Department of Mathematical Sciences
Carnegie Mellon University
Pittsburgh, PA, U.S.A.
giovanni@andrew.cmu.edu

Abstract: A variational model proposed in the physics literature to describe the onset of pattern formation in two-component bilayer membranes and

amphiphilic monolayers leads to the analysis of a Ginzburg-Landau type energy, precisely,

$$u \mapsto \int_\Omega \left[W(u) - q\,|\nabla u|^2 + \left|\nabla^2 u\right|^2 \right] dx.$$

When the stiffness coefficient $-q$ is negative, one expects curvature instabilities of the membrane and, in turn, these instabilities generate a pattern of domains that differ both in composition and in local curvature. Scaling arguments motivate the study of the family of singular perturbed energies

$$u \mapsto F_\varepsilon(u, \Omega) := \int_\Omega \left[\frac{1}{\varepsilon} W(u) - q\varepsilon |\nabla u|^2 + \varepsilon^3 |\nabla^2 u|^2 \right] \, dx.$$

Here, the asymptotic behavior of $\{F_\varepsilon\}$ is studied using Γ-convergence techniques. In particular, compactness results and an integral representation of the limit energy are obtained.